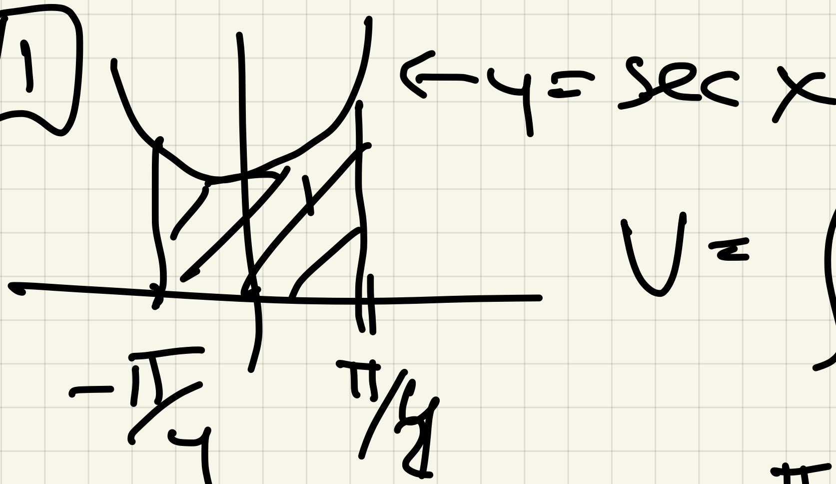
8/14/Calc2

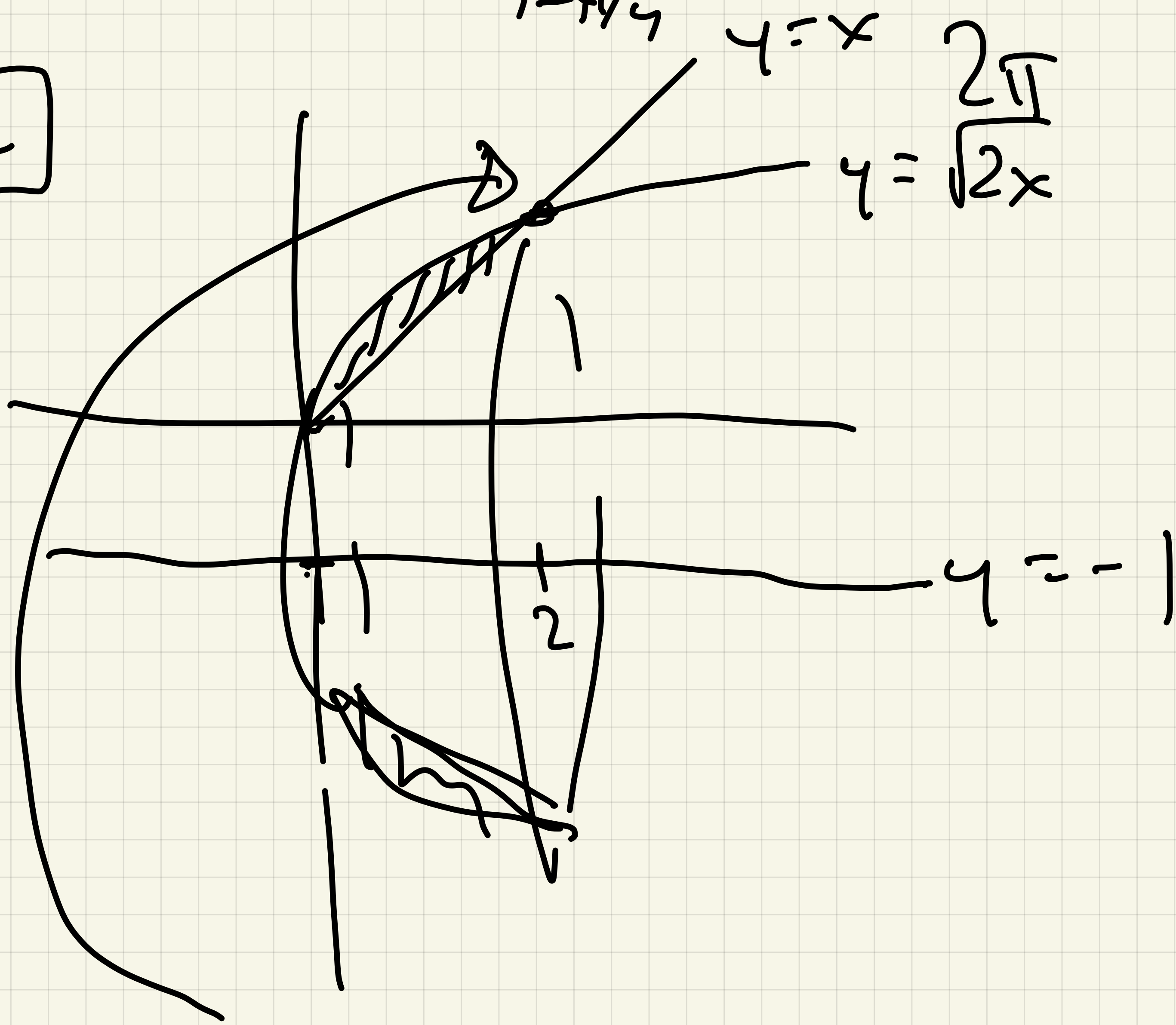
$\boxed{1}$



$$V = \int_{-\pi/4}^{\pi/4} \pi(\sec x)^2\,dx$$

$$\pi \tan x \Big|_{-\pi/4}^{\pi/4} = \pi - (-\pi) = 2\pi$$

$\boxed{2}$



$$\sqrt{2x} = x$$

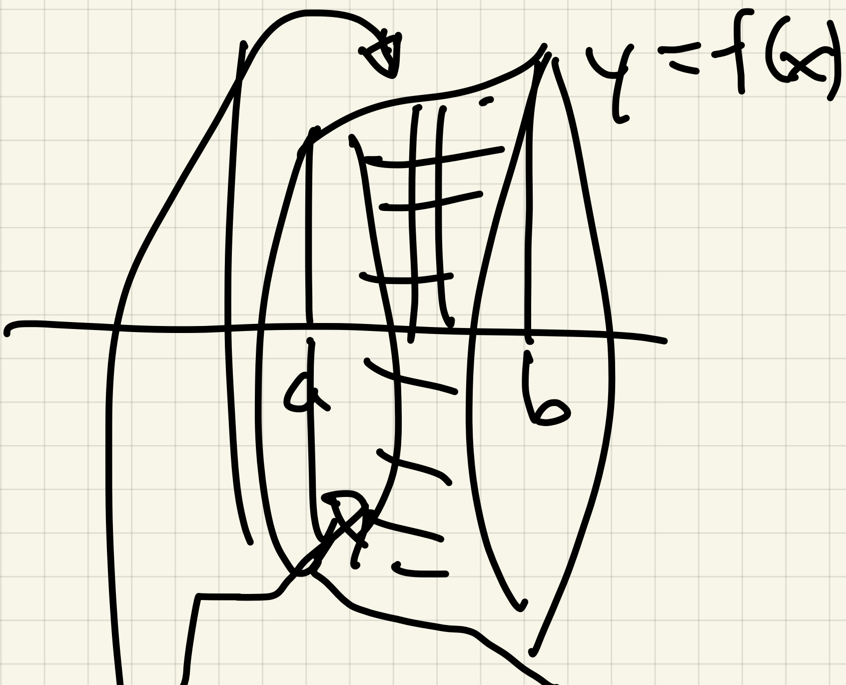
$$2x = x^2 \Rightarrow x = 0, 2$$

$$\int_0^2 \pi \Big(\underbrace{(\sqrt{2x}+1)^2}_{\text{out}} - (x+1)^2 \Big) dx$$

Last time

Arc length

SA of revolution



Arc length

SA revolution

$$ds = \sqrt{1 + (f'(x))^2}\, dx$$

$$L = \int_a^b ds$$

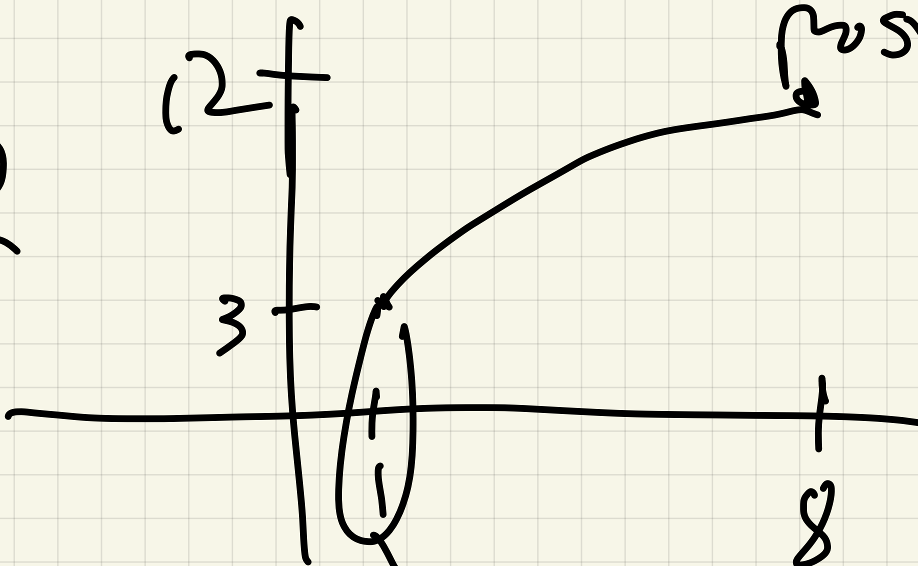
$$SA = \int_a^b 2\pi f(x)\, ds$$

radius of strip at position x.

Ex 0

$y = 3x^{2/3}$

$1 \le x \le 8$



Arc length

$\frac{dy}{dx} = 2x^{-1/3} \Rightarrow$

$$L = \int ds = \int_1^8 \sqrt{1+4x^{-2/3}}\,dx$$

$$= \sqrt{1+\frac{4}{(x^{1/3})^2}}$$

$$\sqrt{\frac{(x^{1/3})^2+4}{(x^{1/3})^2}} = \frac{1}{x^{1/3}}\sqrt{x^{2/3}+4}$$

$$= \int \underline{\underline{x^{-\frac{1}{3}}}} \sqrt{x^{2/3}+4}\; dx$$

$$u = x^{2/3}+4 \qquad x=8,\ u=8$$

$$x=1,\ u=5$$

$$du = \tfrac{2}{3}x^{-1/3}\,dx$$

$$\tfrac{3}{2}\,du = \underline{x^{-1/3}\,dx}$$

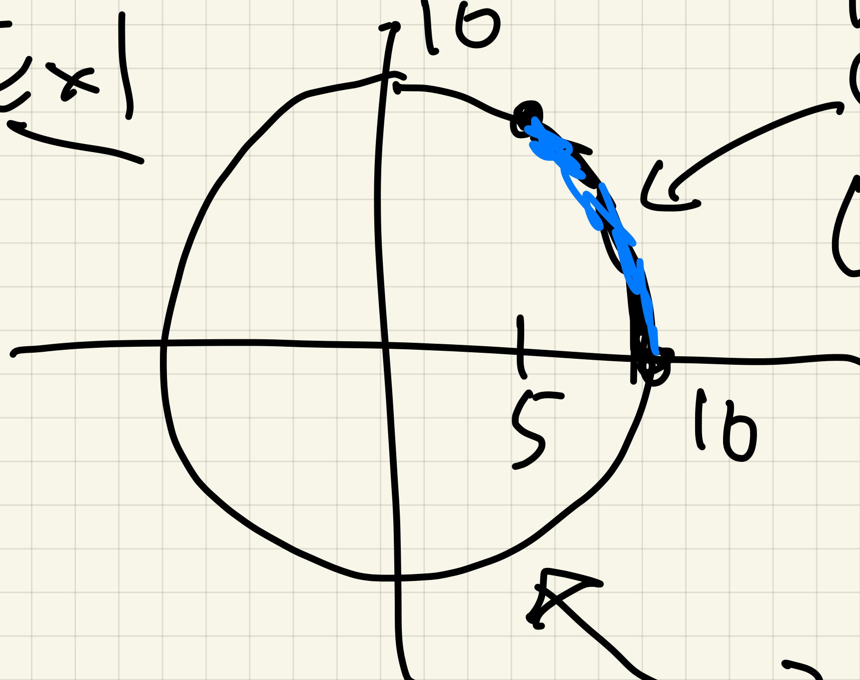
$$\frac{3}{2}\int_5^8 \underline{\underline{\sqrt{u}}}\,du = \cancel{\tfrac{3}{2}}\cdot\cancel{\tfrac{2}{3}}\,u^{3/2}\Big|_5^8 =$$

$$8^{3/2} - 5^{3/2}$$

$$\underline{\underline{SA}}:\ \int_1^8 2\pi\cdot 3x^{2/3}\sqrt{1+4x^{-2/3}}\,dx$$

$$6\pi\int_1^8 x^{2/3}\sqrt{1+4x^{-2/3}}\,dx = ??$$

Ex 1



Find
(a) arc length
(b) SA of revolution about x-axis

$x^2+y^2=100$

(a)

$$y=\sqrt{100-x^2}$$

$$\frac{dy}{dx}=\frac{-\cancel{2}x}{\cancel{2}\sqrt{100-x^2}}$$

$$L=\int ds=\int_5^{10}\sqrt{1+\frac{x^2}{100-x^2}}\,dx$$

$$=\int_5^{10}\sqrt{\frac{100-\cancel{x^2}+\cancel{x^2}}{100-x^2}}\,dx =$$

$$= \int_5^{10} \frac{10}{\sqrt{100-x^2}}\,dx =$$

Recall: $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$

$$10 \arcsin\frac{x}{10}\Big|_5^{10} =$$

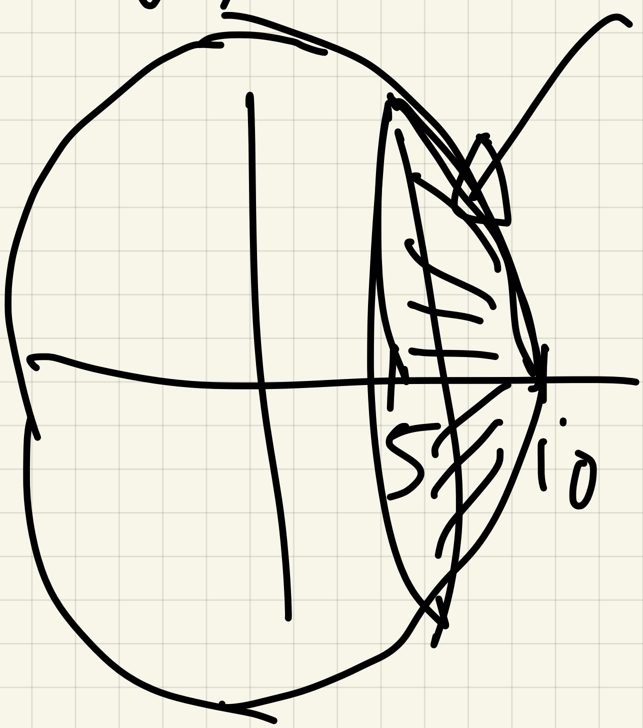
$$10\,\underbrace{\arcsin 1}_{\pi/2} - 10\,\underbrace{\arcsin\frac{5}{10}}_{\pi/6}$$

$$= 10\left(\frac{\pi}{2} - \pi/6\right) = \frac{10\pi}{3}$$

(Confirmation: blue wedge = 1/6 circum of circle =

$$\frac{2\pi \cdot 10}{6} = \frac{10\pi}{3} \checkmark$$)

(b)



$$SA = \int_5^{10} 2\pi\sqrt{100-x^2}\, ds$$

$$\int_5^{10} 2\pi\sqrt{100-x^2} \cdot \frac{10}{\sqrt{100-x^2}}\, dx$$

$$\int_5^{10} 20\pi\, dx = 20\pi x \Big|_5^{10} = 100\pi$$

Cool fact: SA over any middle arc with same x-width has same surface area

(c) Revolve arc about

y-axis

$$\int_5^{10} 2\pi x \frac{10}{\sqrt{100-x^2}}\,dx =$$

r=x

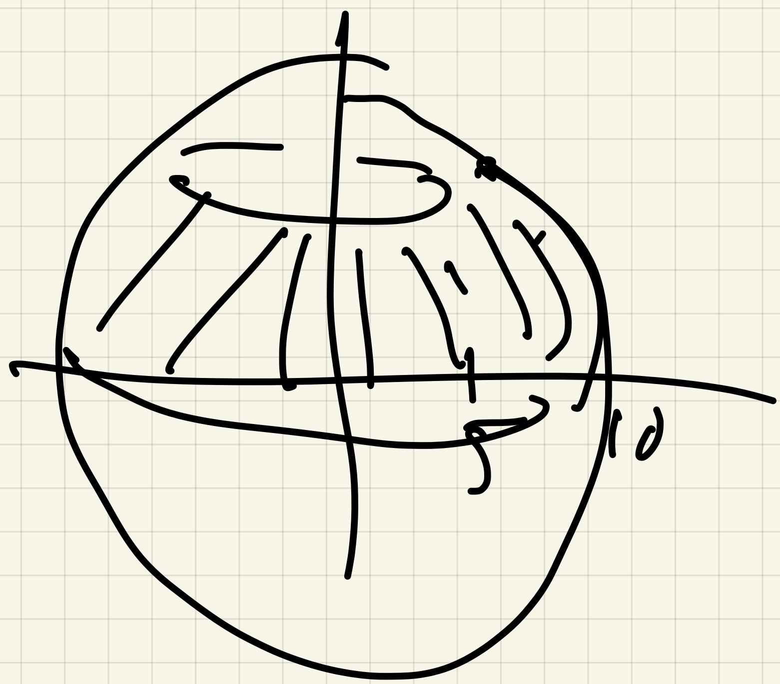
ds

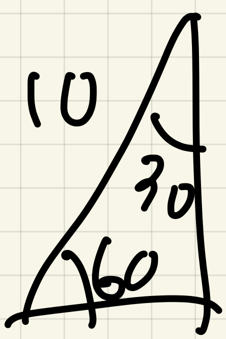
$u=100-x^2$

$$-10\pi\int_5^{10}\frac{-2x}{\sqrt{100-x^2}}\,dx \qquad du=-2x\,dx$$

$\parallel$

$100\pi\sqrt{3}$





$(5) \Rightarrow \quad \frac{10\sqrt{3}}{2}\cdot 20\pi = 100\pi\sqrt{3}$ ✓