

16/5 | Calc 2

Quiz 8

$$\int \underbrace{x^2}_u \underbrace{\sin 5x}_{dv} dx$$

$$du = 2x dx \quad v = -\frac{1}{5} \cos 5x$$

$$-\frac{1}{5} x^2 \cos 5x + \frac{2}{5} \int \underbrace{x}_{u} \underbrace{\cos 5x}_{dv} dx$$

$$du = dx \quad v = \frac{1}{5} \sin 5x$$

$$-\frac{1}{5} x^2 \cos 5x + \frac{2}{5} \left[\frac{1}{5} x \sin 5x - \int \frac{1}{5} \sin 5x \right]$$

$$-\frac{1}{5} x^2 \cos 5x + \frac{2}{25} x \sin 5x + \frac{2}{125} \cos 5x \Big|_0^{\pi/2}$$

$$\frac{\pi}{25} - \frac{2}{125} = \frac{5\pi - 2}{125}$$

Last time: log in finals

$$\textcircled{A} \int \sin^n x \cos^m x \, dx$$

$$n \text{ odd} \Rightarrow u = \cos x$$

$$m \text{ odd} \Rightarrow u = \sin x$$

n, m both even $\Rightarrow$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$\textcircled{B}$ Integrals with $\tan x / \sec x$

$$1 + \tan^2 x = \sec^2 x$$

$$\text{Ex 1} \int \frac{\tan^2 x}{\sec^3 x} \, dx = \int \frac{\sin^2 x \cos^5 x}{\cos^2 x} =$$

$$(c) \int \sin^2 x \cos^3 x \, dx = \quad u = \sin x$$

$$\int \sin^2 x (1 - \sin^2 x) \cos x \, dx$$

$$\int u^2 (1 - u^2) \, du \dots$$

$$(b) \int \tan^5 x \sec^2 x \, dx$$

$u = \tan x$
 $du = \sec^2 x \, dx$

$$\int u^5 \, du = \frac{u^6}{6} + C =$$

$$\frac{\tan^6 x}{6} + C$$

In general, $\int \sec^n x \tan^m x \, dx$
 n even, Then $u = \tan x$

$$(c) \int \sec^n x \, dx =$$

$$\int \sec^2 x \sec^2 x \, dx$$

$$\int (1 + \tan^2 x) \sec^2 x \, dx$$

$$\int (1 + u^2) \, du =$$

$$u + \frac{u^3}{3} + C = \tan x + \frac{\tan^3 x}{3} + C$$

$$(d) \int \tan^3 x \sec^6 x dx =$$

$$\int \tan^3 x \underbrace{\sec^2 x}_{1 + \tan^2 x} \underbrace{\sec^2 x}_{1 + \tan^2 x} \underbrace{\sec^2 x}_{u = \tan x} dx$$

$$\int u^3 (1 + u^2)(1 + u^2) du$$

$$\int u^7 + 2u^5 + u^3 du$$

$$\frac{u^8}{8} + \frac{2}{6} u^6 + \frac{u^4}{4} + C$$

$$\frac{\tan^8 x}{8} + \frac{\tan^6 x}{3} + \frac{\tan^4 x}{4} + C$$

$$(e) \int \tan^7 x dx =$$

$$\int \tan^2 x (\sec^2 x - 1) =$$

$$\int \tan^2 x \sec^2 x - \frac{\tan^2 x}{1} dx$$

$$\int \tan^2 x \sec^2 x - \sec^2 x + 1 dx$$

$$\frac{\tan^3 x}{3} - \tan x + x + C$$

(f) $\int \sec^9 x \tan x dx$

CAN use method above

$$\int (1 + \tan^2 x)^4 \tan x \sec^2 x dx$$

BETTER: $u = \sec x$

$$\int \underbrace{\sec^8 x}_{u^8} \underbrace{\sec x \tan x dx}_{du}$$

$$\frac{u^9}{9} + C$$

$$\frac{\sec^9 x}{90} + C$$

In general $\int \sec^n x \tan^m x dx$
 m odd, $n \geq 0$, then $u = \sec x$
 works

Ex2 $\int \sec^8 x \tan^3 x dx$

(a) $\int \sec^7 x (\sec^2 x - 1) \sec x \tan^2 x dx$

$$\int u^7 (u^2 - 1) du$$

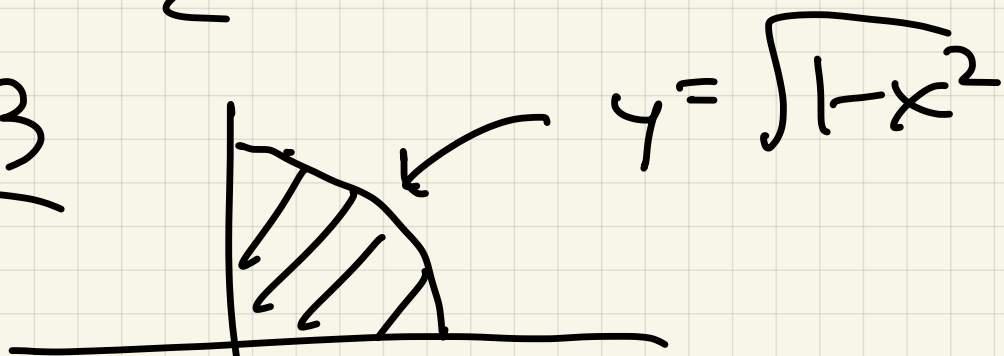
$$\int u^9 - u^7 du$$

$$\frac{u^{10}}{10} - \frac{u^8}{8} + C$$

$$\frac{\sec^{10} x}{10} - \frac{\sec^8 x}{8} + C$$

$$\begin{aligned}
 (b) \quad \int \tan^3 x \, dx &= \\
 \int (\sec^2 x - 1) \tan x \, dx &= \\
 = \int \underbrace{\sec^2 x \tan x}_{\frac{d}{dx} \sec x} - \underline{\underline{\tan x}} \, dx &= \\
 \frac{\sec^2 x}{2} - \ln |\sec x| + C &
 \end{aligned}$$

§ 8.3



Area = $\int_0^1 \sqrt{1-x^2} \, dx$

$x = \sin \theta$

$x = 1 \Rightarrow \theta = \pi/2$

$x = 0 \Rightarrow \theta = 0$

$\frac{dx}{d\theta} = \cos \theta \Rightarrow dx = \cos \theta \, d\theta$

$$\int \sqrt{1 - \sin^2 \theta} \cdot \cos \theta \, d\theta$$

$$\sqrt{\cos^2 \theta}$$

$$\int \cos \theta \cos \theta \, d\theta$$

$$\int \cos^2 \theta \, d\theta =$$

$$\int \frac{1 + \cos 2\theta}{2} \, d\theta$$

$$\left. \frac{1}{2} \theta + \frac{1}{4} \sin 2\theta \right|_{\theta=0}^{\theta=\pi/2} = \frac{\pi}{4}$$