

10/20/Calc2

Quiz 11

$$\textcircled{1} \quad \frac{3x+11}{(x+2)^2(x-3)(x^2+9)} =$$

$$\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3} + \frac{D}{(x-3)} +$$

$$\textcircled{2} \quad \frac{5x^2 - 8x + 33}{(x-1)(x^2+9)} = \frac{\frac{Ex+F}{x^2+9}}{x-1} + \frac{Bx+C}{x^2+9}$$

$$5x^2 - 8x + 33 = A(x^2+9) + (Bx+C)(x-1)$$

$$= (A+B)x^2 + (C-B)x + (9A-C)$$

$$x=1 \Rightarrow 30 = 10A \quad A=3$$

$$B=2$$

$$C=-6$$

$$\text{so } \int \frac{3}{x-1} + \frac{2x-6}{x^2+9} dx$$

$$\int \frac{3}{x-1} + \frac{2x}{x^2+9} - \frac{6}{x^2+9} dx$$

$$3 \ln|x-1| + \ln|x^2+9| - 2 \arctan \frac{x}{3} + C$$

$$\int \frac{1}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a}$$

$$8.7 \quad \int_a^\infty f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$$

Ex 1

$$\int_3^\infty \frac{5x^2 - 8x + 33}{(x-1)(x^2+9)} dx$$

$$\lim_{b \rightarrow \infty} \int_3^b \boxed{} dx =$$

$$\lim_{b \rightarrow \infty} 3 \ln|x-1| + \ln|x^2+9| - 2 \arctan \frac{x}{3} \Big|_3^b$$

$$\lim_{b \rightarrow \infty} \begin{matrix} 3 \ln|b-1| + \ln|b^2+9| - 2 \arctan \frac{b}{3} \\ \downarrow \quad \quad \downarrow \quad \quad \downarrow \\ +\infty \quad \quad \infty \quad \quad -\frac{\pi}{2} \end{matrix} - 3 \ln 2 + \ln 18 + 2 \arctan 1$$

diverges

Special Case:

$$\int_1^{\infty} \frac{dx}{x} - \text{diverges to } +\infty$$

$$\int_1^{\infty} \frac{dx}{x^p} = \lim_{b \rightarrow \infty} \frac{x^{-p+1}}{-p+1} \Big|_1^b =$$

$$\begin{matrix} \downarrow \\ x^{-p} \end{matrix} \lim_{b \rightarrow \infty} \frac{b^{-p+1}}{-p+1} - \frac{1}{-p+1} =$$

$$\lim_{b \rightarrow \infty} \frac{1}{(1-p)b^{p-1}} + \left[\frac{1}{p-1} \right] =$$

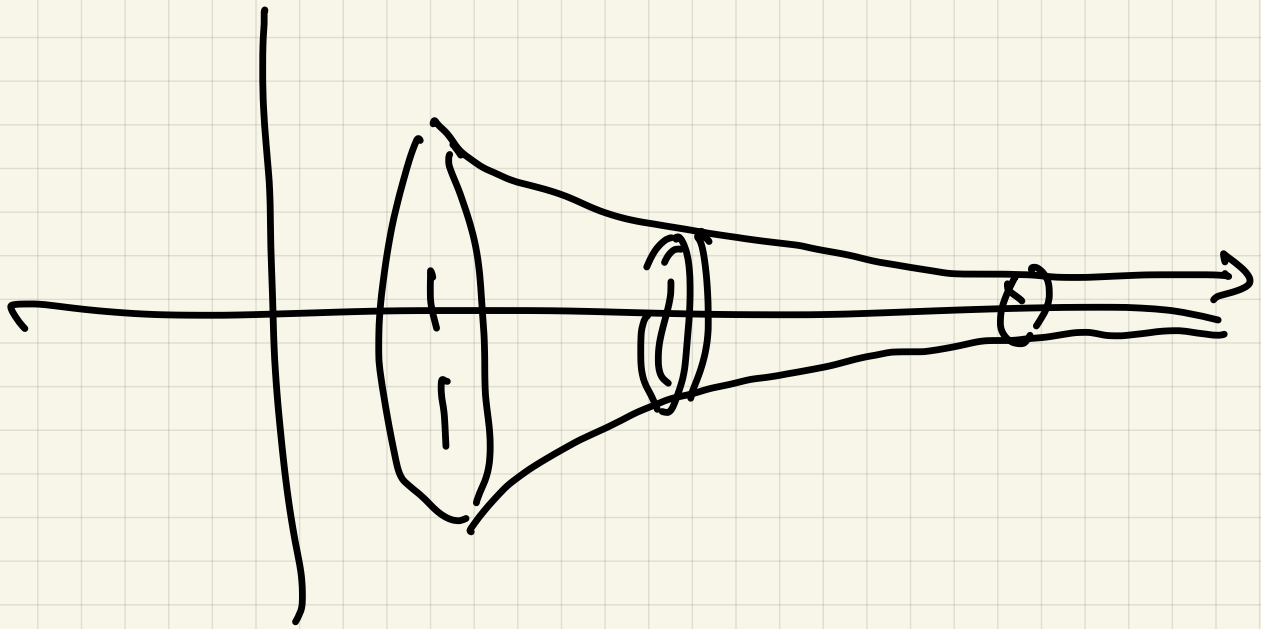
Diverges $\left\{ \begin{array}{l} +\infty \\ \frac{1}{p-1} \end{array} \right.$ $p \leq 1$
 converges to $\frac{1}{p-1}$ $p > 1$

Ex 2 $\int_1^{\infty} \frac{1}{x^2} dx$ converges to 1

Ex 3 $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$ diverges to $+\infty$

Ex 4 Gabriel's Horn:

Revolve the graph of $y = \frac{1}{x}$ about x-axis, $x \geq 1$



Volume inside :

$$Vol = \int_1^{\infty} \pi \left(\frac{1}{x} \right)^2 dx = \pi \int_1^{\infty} \frac{1}{x^2} = \pi$$

$$SA = \int_1^{\infty} 2\pi \left(\frac{1}{x} \right) \sqrt{1 + \left(-\frac{1}{x^2} \right)^2} dx$$

$$\sqrt{\frac{dx}{dx}} = \frac{1}{x^2}$$

$$\int_1^{\infty} 2\pi \left(\frac{1}{x} \right) dx = 2\pi \int_1^{\infty} \frac{1}{x} dx \quad p > 1$$

diverges to $+\infty$.

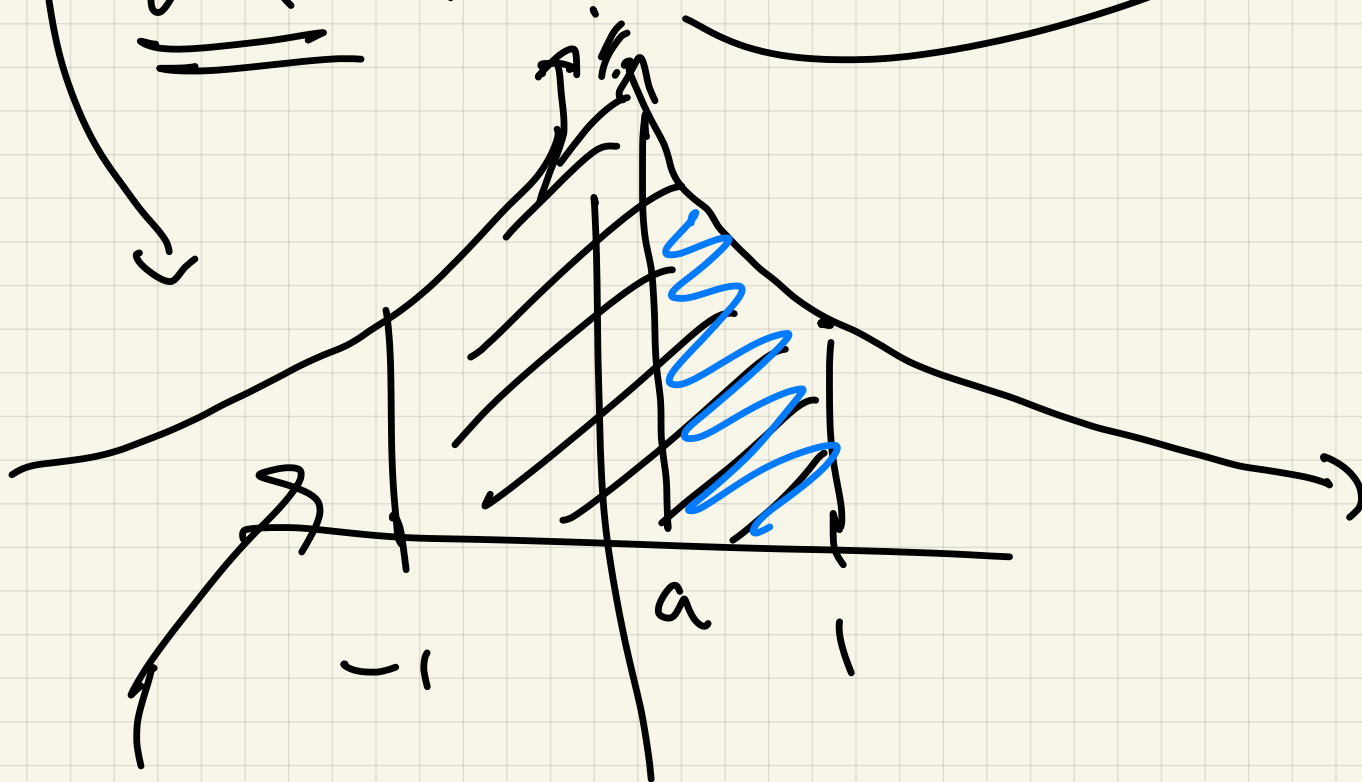
Can fill with parity but can't paint it.

Ex 5

$$\int_{-1}^1 \frac{dx}{x^2} =$$

$$-\frac{1}{x} \Big|_{-1}^1 = -1 - \left(-\frac{1}{-1}\right) = -2$$

What??



region is vertically
improper

Carefully:

$$\int_{-1}^0 \frac{1}{x^2} dx$$

$$\int_0^1 \frac{1}{x^2} dx$$

$$\lim_{a \rightarrow 0^+} \int_a^1 \frac{1}{x^2} dx =$$

$$\lim_{a \rightarrow 0^+} -\frac{1}{x} \Big|_a^1 = \lim_{a \rightarrow 0^+} -\frac{1}{1} - \left(-\frac{1}{a}\right) = \infty$$

So correct answer:
Integral diverges

§ 9.1

Sequences:

ordered list of numbers
called terms

Ex) 2 4 6 8 10 12 14...

Notation: n^{th} term is denoted

a_n or b_n
↑
subscript,
index

Ex1 above

$$a_1 = 2$$

$$a_2 = 4$$

$$a_3 = 6$$

⋮

$$b_0 = 2$$

$$b_1 = 4$$

$$b_2 = 6$$

⋮

More efficient to use formulas

$$a_n = 2n, n \geq 1$$

$$b_n = 2(n+1), n \geq 0$$

Ex 2

$$\left\{ a_n = \frac{n}{n+1} \right\}_{n \geq 1}$$

$$\frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5} \quad \frac{5}{6} \quad \dots$$

$$\{b_n = 2^n\}_{n \geq 0}$$

$$1, 2, 4, 8, 16, 32, \dots$$

Defn: A sequence $\{a_n\}$ has
limit L if we can make

a_n as close to L as we like by choosing n large enough

(For each $\varepsilon > 0$, there is a number N : $n \geq N \Rightarrow$
 $|a_n - L| < \varepsilon$)

Notation: $\lim_{n \rightarrow \infty} a_n = L$ (converges)
or $a_n \rightarrow L$ (diverges)

Ex 1 $a_n = \frac{n}{n+1}$ converges to 1

Note: Can use familiar calc limits.

$$(a) \lim_{n \rightarrow \infty} \frac{n}{n+1} = \lim_{x \rightarrow \infty} \frac{x}{x+1} \stackrel{Lt}{=} 1$$

$$\lim_{x \rightarrow \infty} \frac{1}{1} = 1 \quad \checkmark$$

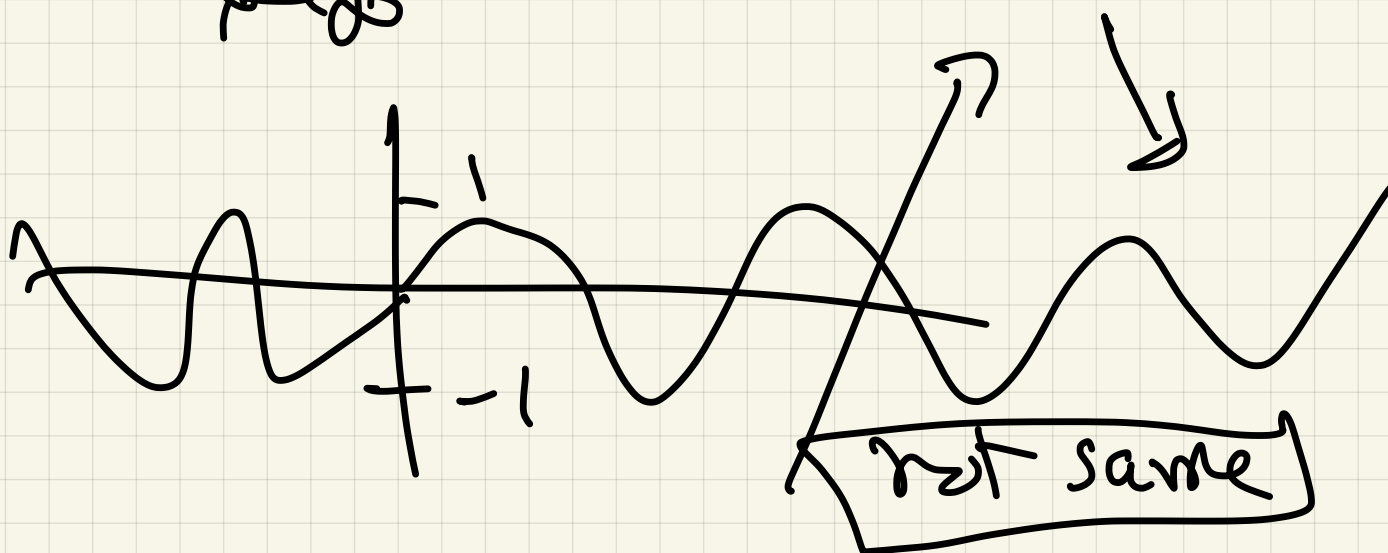
$$(b) \quad \lim_{n \rightarrow \infty} \frac{n^3 - n^2 + 5}{3n^3 + 7} =$$

$$\lim_{x \rightarrow \infty} \frac{x^3 - x^2 + 5}{3x^3 + 7} \stackrel{LH}{=} \frac{1}{3}$$

(k) Ex2 Can't always use Calc! :

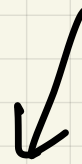
$$(a) \quad \lim_{n \rightarrow \infty} \sin(n\pi)$$

Calc: $\lim_{x \rightarrow \infty} \sin(x\pi)$ DNE



BUT

$$\lim_{n \rightarrow \infty} \sin(n\pi) = 0$$



$$\sin 0$$

$$\sin \pi$$

$$\sin 2\pi$$

$$\sin 3\pi$$