

10/19/Calc2

Last time

Improper integrals

Definitions:

$$① \int_a^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$$

$$② \int_{-\infty}^b f(x) dx = \lim_{a \rightarrow -\infty} \int_a^b f(x) dx$$

$$③ \int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^c f(x) dx + \int_c^{\infty} f(x) dx$$

c any number
Integral converges, if limit exists,
otherwise diverges

Convergence: mean area under

Ex) is bounded - but

$$\int_{-\infty}^{\infty} x dx = \lim_{b \rightarrow \infty} \int_{-b}^b x dx = 0$$



but, want to

diverge

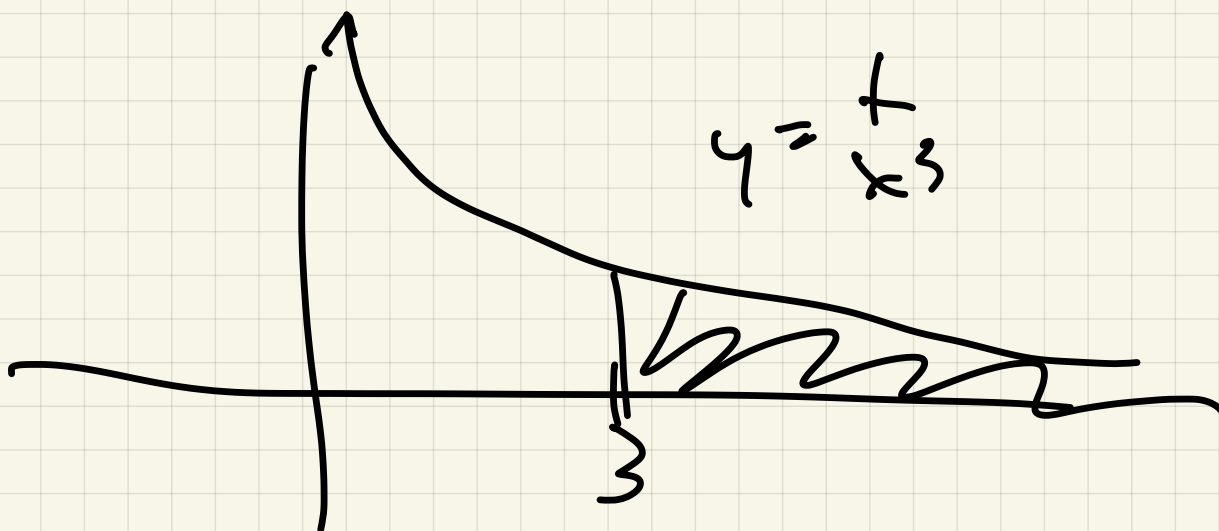
$$\left(\int_0^{\infty} x dx \quad \int_{-\infty}^0 x dx \text{ both diverges} \right)$$

ex2

(c)

$$\int_3^{\infty} \frac{1}{x^3} = \lim_{b \rightarrow \infty} \int_3^b \frac{1}{x^3} dx$$

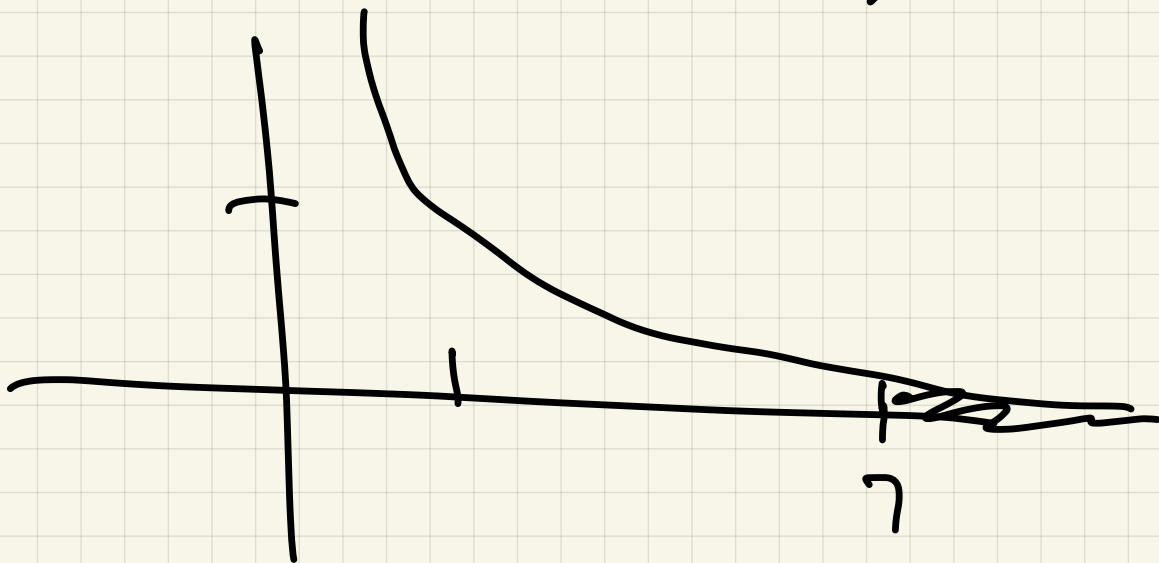
$$\lim -\frac{1}{2x^2} \Big|_3^b = \lim -\frac{1}{2b^2} + \frac{1}{18} = \frac{1}{18}$$



$$(b) \int_7^{\infty} \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow \infty} 2\sqrt{x} \Big|_7^b =$$

$$\lim_{b \rightarrow \infty} 2\sqrt{b} - 2\sqrt{7} = +\infty$$

diverges



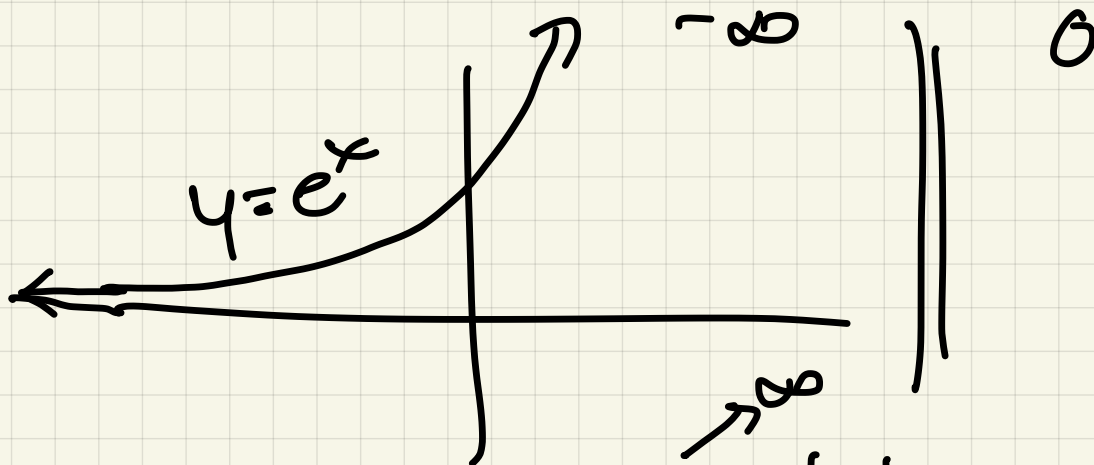
$$(c) \int_{-\infty}^5 x e^x dx =$$

$$\lim_{a \rightarrow -\infty} \int_a^5 x e^x dx =$$

$$\lim_{a \rightarrow -\infty} x e^x - e^x \Big|_a^5 =$$

$$\lim_{a \rightarrow -\infty} 4e^5 - (a-1)e^a$$

$\downarrow$ $\downarrow$



$$4e^5 - \lim_{a \rightarrow -\infty} \frac{a-1}{e^{-a}} = 4e^5 - \lim_{a \rightarrow -\infty} \frac{1}{e^{-a}} = 4e^5 - \lim_{a \rightarrow -\infty} \frac{1}{0} = 4e^5 - \infty = -\infty$$

Ex 3

$$\int_{-\infty}^{\infty} \frac{-2x+20}{3x^3+7x^2+10x} dx$$

$$\frac{-2x+20}{x(x+2)(x+5)} = \boxed{\frac{A}{x} + \frac{B}{x+2} + \frac{C}{x+5}}$$

$$-2x+20 = A(x+2)(x+5) + B(x)(x+5) +$$

$$C x(x+2)$$

$$x=0 \Rightarrow 20 = 10A \quad A=2$$

$$x=-2 \Rightarrow 24 = -6B \quad B=-4$$

$$x = -5 \Rightarrow 30 = 15^\circ\text{C} \quad C = 2$$

$$\int_3^\infty dx =$$

$$\lim_{b \rightarrow \infty} \int_3^b \left(\frac{2}{x} - \frac{9}{x+2} + \frac{2}{x+5} \right) dx$$

$$\lim_{b \rightarrow \infty} \left(2 \ln|x| - 9 \ln|x+2| + 2 \ln|x+5| \right) \Big|_3^b$$

$$\lim_{b \rightarrow \infty} \left(2 \ln b - 9 \ln(b+2) + 2 \ln(b+5) \right) - \left(2 \ln 3 - 9 \ln 5 + 2 \ln 8 \right)$$

$$\lim_{b \rightarrow \infty} \ln \frac{b^2 (b+5)^2}{(b+2)^9} - \ln 9 + \ln 5^9 - \ln 8^2$$

$$\lim_{b \rightarrow \infty} \ln \frac{1 \left(1 + \frac{5}{b}\right)^2}{\left(1 + \frac{2}{b}\right)^9} + \ln \frac{5^9}{9 \cdot 64}$$

$$0 + \ln \frac{625}{576}$$