

10/15/ Calc 2

Quiz 10

[1]

$$\int \frac{x}{(\sqrt{25+x^2})^3} dx$$

$$x = 5 \tan \theta$$

$$dx = 5 \sec^2 \theta d\theta$$

$$\sqrt{25+x^2} = 5 \sec \theta$$

$$\int \frac{5 \tan \theta}{5^3 \sec^3 \theta} 5 \sec^2 \theta d\theta =$$

$$\frac{1}{5} \int \frac{\tan \theta}{\sec \theta} d\theta = \frac{1}{5} \int \sin \theta d\theta =$$

$$-\frac{1}{5} \cos \theta + C$$

[2]

$$\int \frac{dx}{(\sqrt{9-x^2})^3}$$

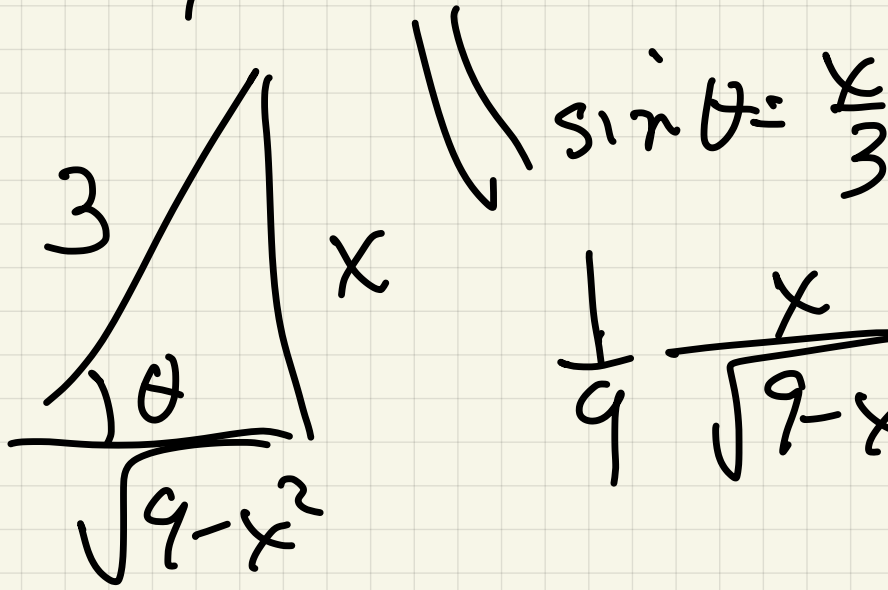
$$x = 3 \sin \theta$$

$$dx = 3 \cos \theta d\theta$$

$$\sqrt{\quad} = 3 \cos \theta$$

$$\int \frac{3 \cos \theta d\theta}{27 \cos^3 \theta} = \frac{1}{9} \int \sec^2 \theta d\theta =$$

$$\frac{1}{9} \tan \theta + C$$



$$\frac{1}{9} \frac{x}{\sqrt{9-x^2}} + C$$

Last time: Partial fractions

$$\text{Exo} \int \frac{8x^3+4}{x^4-1} = \int \frac{8x^3+4}{(x^2+1)(x-1)(x+1)}$$

$$\frac{A}{x-1} + \frac{B}{x+1} + \frac{C(x+1)}{x^2+1}$$

$$8x^3+4 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + C(x+1)(x^2-1)$$

$$8x^3 + 4 = (A+B+C)x^3 + (A-B+D)x^2 + (A+B-C)x + (A-B-D)$$

$$(1) A+B+C=8$$

$$(2) A-B+D=0$$

$$(3) A+B-C=0$$

$$(4) A-B-D=4$$

$$x=1 \quad 12=4A$$

$$x=-1 \quad -4=-4B$$

$$A=3, B=1$$

$$(3) C=A+B=4$$

$$(4) D=A-B-4=-2$$



$$\int \frac{3}{x-1} + \frac{1}{x+1} + \frac{4x}{x^2+1} - \frac{2}{x^2+1} dx$$

$u = x^2 + 1$

$$= 3 \ln|x-1| + \ln|x+1| + 2 \ln|x^2+1| - 2 \arctan x + C$$

§ 4.5 (Bernoulli) $\lim_{x \rightarrow c} \frac{f}{g}$

L'Hospital's rule:

If $\left[\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x) = 0 \right]$, Then

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)} \quad \left(\text{if this exists} \right)$$

Ex) (a) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{3x^2}{2x} = 3\checkmark$

$$\lim_{x \rightarrow 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} = \frac{12}{4} = 3$$

$$(b) \lim_{x \rightarrow 0} \frac{x}{e^{5x} - 1} = \lim_{x \rightarrow 0} \frac{1}{5e^{5x}} = \frac{1}{5}$$

$$(c) \lim_{x \rightarrow 1} \frac{x}{e^{5x} - 1} = \frac{1}{e^5 - 1}$$

$$(d) \lim_{x \rightarrow 0} \frac{\sin 3x}{5x} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{3 \cos 3x}{5} = \frac{3}{5}$$

Note: L'Hospital's rule also

applies when

$$\lim f = \pm \infty$$

$$\lim g = \pm \infty$$

Ex 2

$$(a) \lim_{x \rightarrow \infty} \frac{3x+7}{6x+12} = \lim_{x \rightarrow \infty} \frac{3}{6} = \frac{1}{2}$$

$$(b) \lim_{x \rightarrow \infty} \frac{2x}{3x^2+5} = \lim_{x \rightarrow \infty} \frac{2}{6x} = 0$$

L'H

$$(c) \lim_{x \rightarrow \infty} \frac{x^3}{\sqrt{x^6+2}} =$$

L'H L'H

$$\lim_{x \rightarrow \infty} \frac{3x^2}{\frac{6x^5}{2\sqrt{x^6+2}}} =$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^6+2}}{x^3} \stackrel{LH}{=} \lim_{x \rightarrow \infty} \frac{x^3}{\sqrt{x^6+2}}$$

??

$$\lim_{x \rightarrow \infty} \sqrt{\frac{x^6}{x^6+2}} =$$

$$= \sqrt{\lim_{x \rightarrow \infty} \frac{x^6}{x^6+2}} = \sqrt{\frac{\cancel{6x^6}}{\cancel{6x^6} + 2}} = \sqrt{1} = 1$$

Other indeterminate forms

LHS: $\frac{0}{0}$ $\frac{\neq \infty}{\neq \infty}$

(A) $0 \cdot \infty$

(B) " 1^∞ "

(C) " 0^0 "

(A) " $0 \cdot \infty$ "

Idea: rewrite as fraction

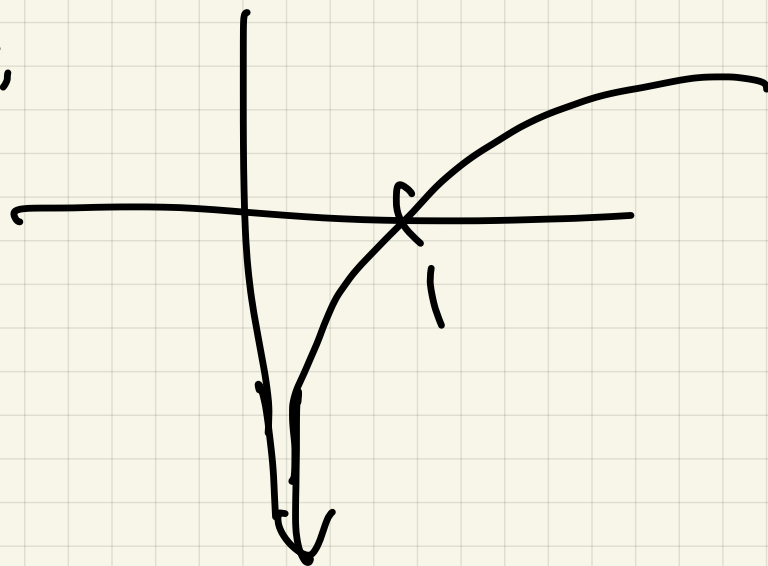
$$\lim_{x \rightarrow \infty} \frac{1}{x} \cdot 3x = 3$$

(c) $\lim_{x \rightarrow 0^+} x \ln x$

// trick:

$$\lim_{x \rightarrow 0^+} \frac{\ln x}{\left(\frac{1}{x}\right)}$$

// LH



$$\lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{1}{x^2}} = \lim_{x \rightarrow 0^+} -x = 0$$

$$(b) \lim_{x \rightarrow 0^+} x e^{1/x} = \lim_{x \rightarrow 0^+} \frac{e^{1/x}}{1/x} = \infty$$

$$\lim_{x \rightarrow 0^+} \frac{-\frac{1}{x^2} e^{1/x}}{-\frac{1}{x^2}} = \lim_{x \rightarrow 0^+} e^{1/x} = \infty$$

Ⓑ + Ⓒ Idea: Take logarithm

$$(a) L = \lim_{x \rightarrow 0^+} x^x$$

$$\ln L = \lim_{x \rightarrow 0^+} \ln x^x$$

$$= \lim_{x \rightarrow 0^+} x \ln x = 0$$

$$\ln L = 0 \Rightarrow L = e^0 = 1.$$

$$(b) L = \lim_{x \rightarrow 0^+} (1+2x)^{1/x}$$

$$\text{So } \ln L = \lim_{x \rightarrow 0^+} \ln(1+2x)^{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{x} \ln(1+2x)$$

$$\approx \lim_{x \rightarrow 0^+} \frac{\ln(1+2x)}{x} \quad \text{L'H}$$

$$\lim_{x \rightarrow 0^+} \frac{\frac{2}{1+2x}}{1} = 2 = \ln L$$

$$L = e^2$$

$$(c) \quad L = \lim_{x \rightarrow \infty} \left(1 - \frac{3}{x}\right)^x$$

$$\ln L = \lim_{x \rightarrow \infty} \ln \left(1 - \frac{3}{x}\right)^x$$

$$= \lim_{x \rightarrow \infty} x \ln \left(1 - \frac{3}{x}\right)$$

$\infty \cdot 0$

$$\lim_{x \rightarrow \infty} \frac{\ln(1 - \frac{3}{x})}{\frac{1}{x}} = LH$$

$$\lim_{x \rightarrow \infty} \frac{\frac{1}{(-\frac{3}{x})} \cdot \frac{+3}{x^2}}{-\frac{1}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \frac{-3}{1 - \frac{3}{x}} = -3$$

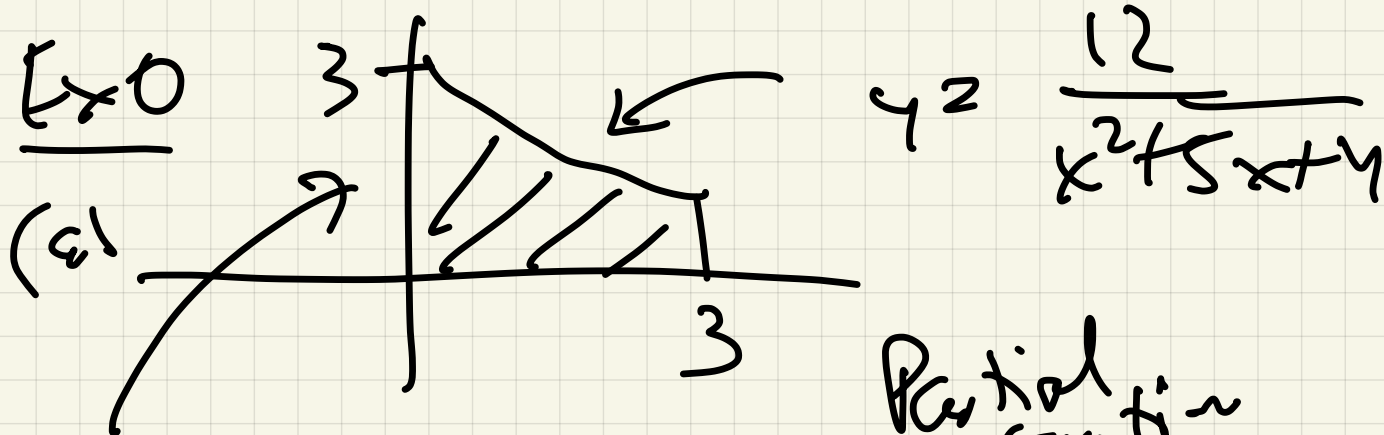
$$\text{So } L = e^{-3} = \frac{1}{e^3}$$

ASIDE:
 Finance
 formula

$$P(1 + \frac{r}{n})^{nt}$$

P = prin
 r = rat
 n =

§ 8.7 Improper Integrals



Partial fraction

Area = $\int_0^3 \frac{12}{(x+1)(x+4)} dx =$

$$\int_0^3 \frac{4}{x+1} - \frac{4}{x+4}$$

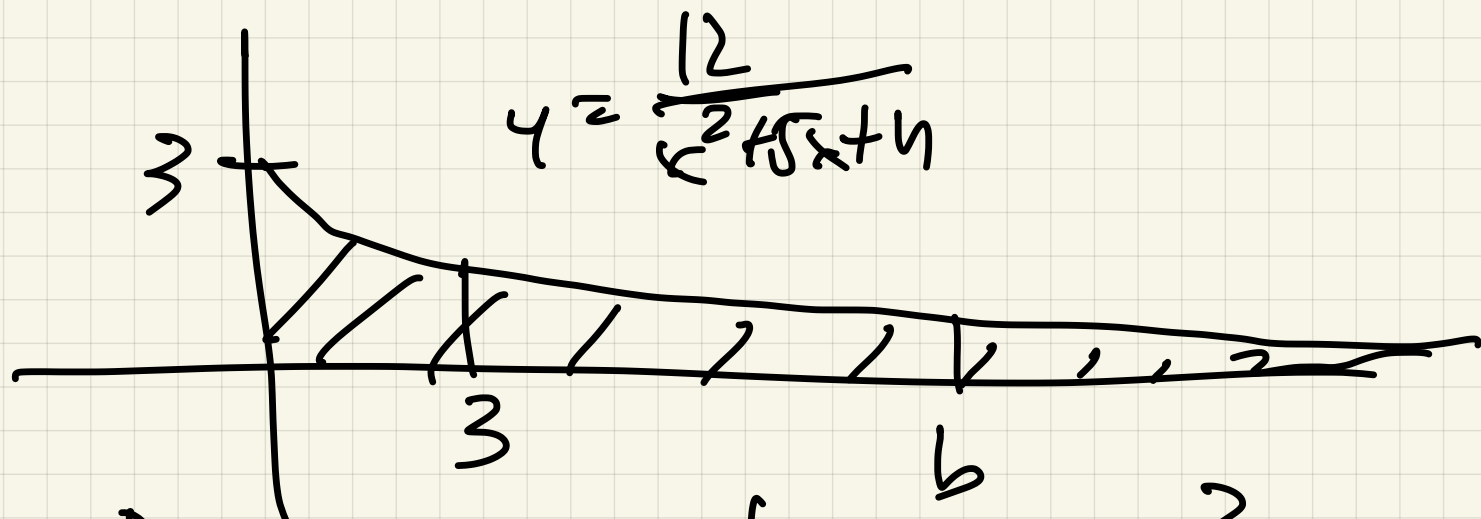
$$4 \ln |x+1| - 4 \ln |x+4| =$$

$$\left(4 \ln \left| \frac{x+1}{x+4} \right| \right) \Big|_0^3$$

$$4 \ln \frac{4}{7} - 4 \ln \frac{1}{4} =$$

$$4 \ln \left(\frac{4}{1/4} \right) = 4 \ln \frac{16}{7}$$

(b) Find the area



Does area make sense?

USE LIMITS:

$$\text{Area} = \lim_{b \rightarrow \infty} \int_0^b \frac{12}{x^2 + 5x + 4} dx =$$

$$\lim_{b \rightarrow \infty} 4 \ln \left(\frac{b+1}{b+4} \right) = 4 \ln \frac{1}{4}$$

$$\ln 1 = 0$$

$$-4 \ln \frac{1}{4}$$

$$= 4 \ln 4$$

$$= \ln 4^4 = \ln 256$$