

10/13/Calc2

Quiz 9 avg 83
med 100

1) $\int \sin^3 x \cos^6 x dx$

$$\int (1 - \cos^2 x) \cos^5 x \sin x dx$$

$$u = \cos x \\ du = -\sin x dx$$

$$-\int (1 - u^2) u^5 du = \int u^7 - u^5 du \\ \frac{u^8}{8} - \frac{u^6}{6} + C = \frac{\cos^8 x}{8} - \frac{\cos^6 x}{6} + C$$

2) $\int \sec^5 x \tan^3 x dx =$

$$\int \sec^4 x (\sec^2 x - 1) \sec x \tan x dx$$

$$u = \sec x$$

$$\int u^4 (u^2 - 1) du = \int u^6 - u^4 du =$$

$$\frac{u^7}{7} - \frac{u^5}{5} + C = \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$$

Last time Partial fractions:

$$\int \frac{N(x)}{D(x)} dx \quad \underline{\deg N < \deg D}$$

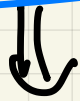
- ① Factor $D(x)$ into linear/quadratic factors
- ② Write form of partial fraction
- ③ Solve for constants
- ④ Integrate result

Ex 6 $\int \frac{2x^4 - 4x^3 + x + 4}{x^4 - 2x^3}$

long division $\Rightarrow$

$$2 + \frac{x+4}{x^4 - 2x^3}$$

$$\frac{x+4}{x^3(x-2)^0} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-2}$$



$$x+4 = Ax^2(x-2) + Bx(x-2) + C(x-2) + D(x^3)$$

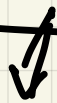
$$1x+4 = (A+D)x^3 + (-2A+B)x^2 + (-2B+C)x + (-2C)$$

$$A+D = 0$$

$$-2A+B = 0$$

$$-2B+C = 1$$

$$-2C = 4$$



x^3

x^2

x

const

$$-2B = 3$$

$$C = -2, B = -3/2, A = -3/4$$

$$D = 3/4$$

FINALLY,

$$\int \frac{2x^4 - 7x^3 + x + 4}{x^7 - 2x^3} dx =$$

$$\int 2 + \frac{-3/4}{x} + \frac{-3/2}{x^2} + \frac{-2}{x^3} + \frac{3/4}{x-2} dx$$

$$= 2x - 3/4 \ln|x| + \frac{3/2}{x} + \frac{1}{x^2} + 3/4 \ln|x-2| + C$$

Ex 1

$$\int \frac{6x^2 + x - 10}{x^3 + 5x} dx$$

$x^2 - 5 = (x - \sqrt{5})(x + \sqrt{5})$

$x'(x^2 + 5)$

does not factor,

(b/c $x^2 + 5 = 0$ has no solution)

$$\frac{6x^2 + x - 10}{x^3 + 5x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 5}$$

$$6x^2 + x - 10 = A(x^2 + 5) + (Bx + C)x$$

$$6x^2 + x - 10 = \underbrace{(A+B)}_6 x^2 + \underbrace{C}_1 x + \underbrace{(5A)}_{-10}$$

$$A = -2, C = 1,$$

$$\left. \begin{array}{l} A+B=6 \\ A=-2 \end{array} \right\} \Rightarrow B=8$$

$$\int \frac{-2}{x} + \frac{8x+1}{x^2+5} dx =$$

$$\int \frac{-2}{x} + \frac{8x}{x^2+5} + \frac{1}{x^2+5} dx$$

$$= -2 \ln|x| + 4 \ln|x^2+5| +$$

$$\frac{1}{\sqrt{5}} \arctan \frac{x}{\sqrt{5}} + C$$

Aside: How to tell if

$ax^2 + bx + c$ factors?

i.e. when does $ax^2 + bx + c = 0$
have solutions?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

key: $\begin{cases} b^2 - 4ac \geq 0 \rightarrow \text{factors} \\ b^2 - 4ac < 0 \rightarrow \text{no factoring} \end{cases}$

Ex 2 (a) $x^2 + 1 = x^2 + 0x + 1$
not (NO)

(b) $x^2 + 3x + 2$

$$b^2 - 4ac = 9 - 4(1)(2) = 1 \geq 0$$

factors: $(x+2)(x+1) \checkmark$

(c) $x^2 + 3x + 1$

$$b^2 - 4ac = 9 - 4 = 5 > 0$$

factors:

$$\left(x + \frac{3+\sqrt{5}}{2}\right) \left(x + \frac{3-\sqrt{5}}{2}\right)$$

$$(b) \quad x^2 + 3x + 3$$

$$b^2 - 4ac = -3 < 0$$

(no).

Ex 3 Write form of the partial fraction:

$$(a) \quad \frac{1}{x^4 - 1} = \frac{1}{(x^2 + 1)(x^2 - 1)} =$$

$$\frac{1}{(x^2 + 1)(x - 1)(x + 1)} =$$

$$\frac{Ax + B}{x^2 + 1} + \frac{C}{x - 1} + \frac{D}{x + 1}$$

$$(b) \quad \frac{1}{x^8 - 64x^2} = \frac{1}{x^2(x^6 - 64)} =$$

$$\frac{1}{x^2(x^3-8)(x^3+8)} =$$

$$\frac{1}{x^2(\underline{x-2})(\underline{x^2+2x+4})(\cancel{x+2})(x^2-2x+4)}$$

↓ form

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{Dx+E}{x^2+2x+4} + \frac{F}{x+2} +$$

$$\frac{Gx+H}{x^2-2x+4}$$

(c) $\int \frac{1}{(x^2+2x-3)^2 (x^2+2x+3)^2}$

↗

$$\frac{((x+3)(x-1))^2 (x^2+2x+3)^2}{\parallel \text{form}}$$

$$\frac{A}{x+3} + \frac{B}{(x+3)^2} = \frac{C}{x-1} + \frac{D}{(x-1)^2} +$$

$$\frac{Ex+F}{x^2+2x+3} + \frac{Gx+H}{(x^2+2x+3)^2}$$

Ex 4 $\int \frac{8x^3+4}{x^4-1} = (x-1)(x+1)(x^2+1)$

$$\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

$$8x^3+4 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x^2-1)$$

$$8x^3+4 = (A+B+C)x^3 + (A-B+D)x^2 + (A+B-C)x + (A+B-D)$$

$$A+B+C = 8$$

$$A - B + D = 0$$

$$A + B - C = 0$$

$$A + B - D = 4$$