

10/12/ Calc2

Last time

trig substitution

$$\sqrt{a^2 - x^2} \Rightarrow x = a \sin \theta$$

$$\sqrt{a^2 + x^2} \Rightarrow x = a \tan \theta$$

$$\sqrt{x^2 - a^2} \Rightarrow x = a \sec \theta$$

Ex 0

$$\int \frac{2}{x^2 - 1}$$

$$x = \sec \theta$$

$$2 \int \csc \theta d\theta = -2 \ln |\csc \theta + \cot \theta|$$

$$\dots = \ln \left| \frac{x-1}{x+1} \right| + C$$

$$\ln |x-1| - \ln |x+1| + C$$

Better way:

$$\text{Observe: } \frac{2}{x^2 - 1} = \frac{2}{(x-1)(x+1)} =$$

How

$$\frac{1}{x-1} \quad \frac{1}{x+1}$$

$$\therefore \int \frac{1}{x-1} - \frac{1}{x+1} dx = \ln|x-1| - \ln|x+1| + C$$

Key step: recognise ~~the~~
how to decompose into
simple pieces

Ex1 $\int \frac{3x^2 + 5x + 9}{x^3 + 4x} dx =$

$$\int \frac{3x^2 + 5x + 9}{x(x^2 + 4)} dx$$

|| MAGIC

$$\int \frac{2x}{x^2 + 4} + \frac{5}{x^2 + 4} + \frac{1}{x} dx$$

$$= \ln|x^2+4| + \frac{5}{2} \arctan \frac{x}{2} + \ln|x| + C$$

Method of partial fractions

Want to integrate $\int \frac{N(x)}{D(x)} dx$

- ① Factor $N(x)$ into powers of linear factors and quadratic factors
- $(px+q)^m$ $(ax^2+bx+c)^n$
- ↑
Ans given below

② Form of partial fractions

- For each $(px+q)^m$ include

$$\frac{A_1}{px+q} + \frac{A_2}{(px+q)^2} + \dots + \frac{A_m}{(px+q)^m}$$

- For each $(ax^2+bx+c)^n$

$$\frac{B_1x+C_1}{(ax^2+bx+c)^1} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \dots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}$$

③ Solve for A_i, B_i, C_i ,
 real numbers
 and integrants.

Ex 2 $\int \frac{2x+1}{x^3+3x^2+2x} dx =$

$$\int \frac{2x+1}{x(x^2+3x+2)} = \int \frac{2x+1}{x(x+1)(x+2)}$$

$$\frac{2x+1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$$

$$2x+1 = A(x+1)(x+2) + Bx(x+2) + Cx(x+1)$$

$$x=0 \Rightarrow 1 = A \cdot 1 \cdot 2 \Rightarrow A = \frac{1}{2}$$

$$x=-1 \Rightarrow -1 = B(-1) \Rightarrow B = 1$$

$$x = -2 \Rightarrow -3 = C(-2)(-1) \Rightarrow C = -3/2$$

$$\int \frac{2x+1}{x(x+1)(x+2)} dx = \int \frac{\frac{1}{2}}{x} + \frac{1}{x+1} - \frac{3/2}{x+2} dx$$

$$= \frac{1}{2} \ln|x| + \ln|x+1| - \frac{3}{2} \ln|x+2| + C$$

$2x^3$ $\int \frac{2x^4 - 4x^3 + x + 4}{x^4 - 2x^3} dx$

$$\begin{array}{r} x^4 - 2x^3 \overline{) 2x^4 - 4x^3 + x + 4} \\ \underline{2x^4 - 4x^3} \\ x + 4 \end{array}$$

So

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$$\int 2 dx + \int \frac{x+4}{x^4 - 2x^3} dx$$

$$2x + \int \frac{x+7}{x^3(x-2)} dx$$

$$\frac{x+7}{x^3(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-2}$$

solve A, B, C, D