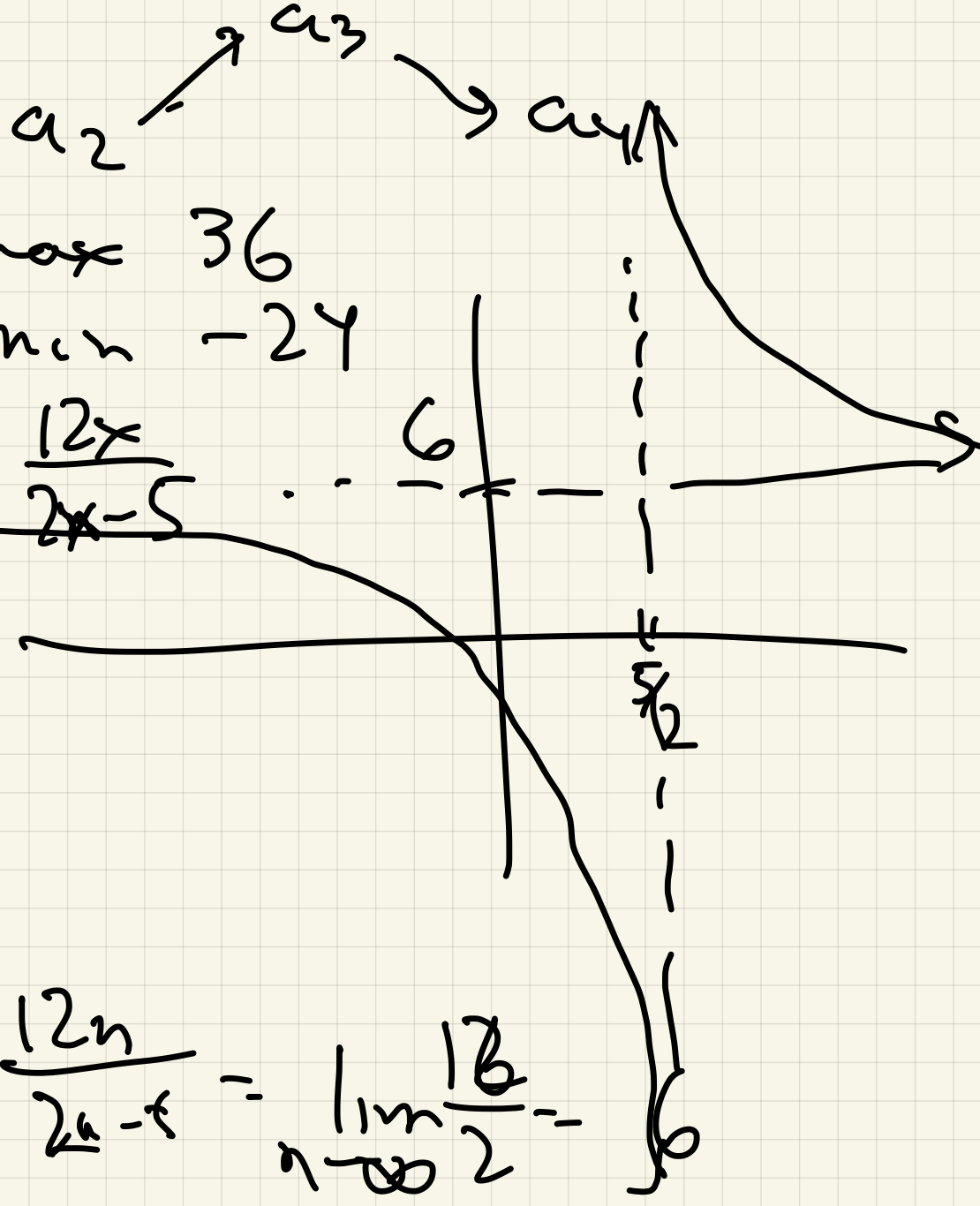
4/3/ Calc2 Quiz 14 avg 93 med 100

$a_n = \frac{12n}{2n-5}$ $n = 1, 2, 3, 4$

① $a_1 = -4$, $a_2 = -24$, $a_3 = 36$, $a_4 = 16$

②



③ 2

max 36

min −24

alt: $f(x) = \frac{12x}{2x-5}$

④ $\lim_{n\to\infty} \frac{12n}{2n-5} = \lim_{n\to\infty} \frac{12}{2} = 6$

Recently: $\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} s_n$

n^{th} term test $\left(\lim_{n\to\infty} a_n \neq 0 \Rightarrow \text{div}\right)$

telescoping series

Geometric series $\sum_{k=0}^{\infty} ar^k$ $\boxed{-1<r<1}$

Integral test

p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $\int_1^{\infty} f$ $\boxed{p>1}$

Ex 0 $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges

b/c $\frac{1}{n(n+1)} \leq \frac{1}{n^2}$

$\sum \frac{1}{n^2}$ conv (p-series $p=2>1$)

partial sums $\sum \frac{1}{n^2}$ bdd $\Rightarrow$

[partial sums $\sum \frac{1}{n(n+1)}$ also bdd]

Direct Comparison Test (DCT)

$0 \le a_n \le b_n$

① $\sum b_n$ converge $\Rightarrow \sum a_n$ converge

② $\sum a_n$ diverges $\Rightarrow \sum b_n$ diverges

Ex (a) $\sum_{n=2}^{\infty} \frac{3^n - 2^n}{5^n}$

Notice $\frac{3^n - 2^n}{5^n} < \frac{3^n}{5^n}$

$\sum \frac{3^n}{5^n}$ converges (geometric $r = 3/5 < 1$)

$\therefore \sum \frac{3^n - 2^n}{5^n}$ converges (by DCT)

(b) $\sum_{n=1}^{\infty} \frac{S_{n-2}}{n^3}$ $\quad \frac{S_{n-2}}{n^3} < \frac{S_n}{n^3} = \frac{5}{n^2}$

$\sum_{n=1}^{\infty} \frac{5}{n^2} = 5 \sum_{n=1}^{\infty} \frac{1}{n^2}$ conv

$p = 2 > 1$

∴ by p-series

$\therefore \sum_{n=1}^{\infty} \frac{S_{n-2}}{n^3}$ converges by DCT,

(c) $\sum_{n=1}^{\infty} \frac{S_{n+2}}{n^3}$

trickier: $\frac{S_n}{n^3}$ smaller

$\frac{S_{n+2}}{n^3} \leq \frac{7n}{n^3}$

$\sum \frac{7n}{n^3} = 7 \sum \frac{1}{n^2}$ ← p-series $p = 2 > 1$

∴ converges

$\therefore \sum \frac{S_{n+2}}{n^3}$ conv by DCT,

(f) $\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}}$ $\ln n \geq 1$ for $n = e$

so for $n \geq e$, $\frac{\ln n}{\sqrt{n}} \geq \frac{1}{\sqrt{n}}$

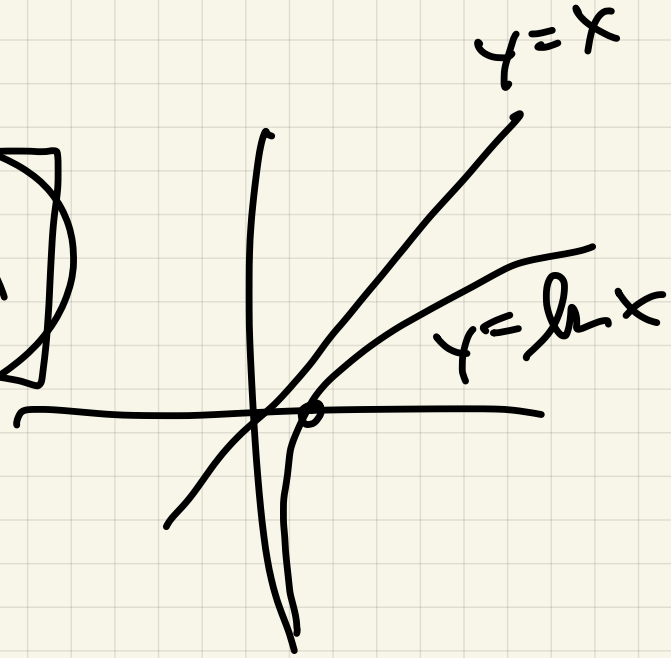
$\sum_{n=3}^{\infty} \frac{1}{\sqrt{n}}$ div by p-series $p = \frac{1}{2} \leq 1$

$\therefore$ DCT $\sum_{n=3}^{\infty} \frac{\ln n}{\sqrt{n}}$ div by DCT

$\therefore \sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}}$ div

(g) $\sum \frac{\ln n}{n^3}$

Observe: $\ln n \leq n$



$\therefore \frac{\ln n}{n^3} \leq \frac{n}{n^3} = \frac{1}{n^2}$

$\sum \frac{1}{n^2}$ conv $(p=2>1)$

$\therefore \sum \frac{\ln n}{n^3}$ conv DCT

(h) $\sum_{n=1}^{\infty} \frac{12}{\sqrt[8]{n^9+17}}$ $\left(\text{expect: conv } \sim \sum \frac{1}{n^{9/8}}\right)$

$$\frac{12}{\sqrt[8]{n^9+17}} \leq \frac{12}{\sqrt[8]{n^9}} = \frac{12}{n^{9/8}}$$

$12 \sum_{n=1}^{\infty} \frac{1}{n^{9/8}}$ conv p-series $p = 9/8 > 1$ ✓

$\therefore \sum_{n=1}^{\infty} \frac{12}{\sqrt[8]{n^9+17}}$ conv by DCT.

(i) $\sum_{n=2}^{\infty} \frac{12}{\sqrt[8]{n^9-17}}$ trickier

$\left(\sim \sum \frac{12}{n^{9/8}} \text{ conv}\right)$

Limit Comparison test (LCT)

If $0 < a_n, b_n$ for all $n \geq N$

① $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$ then

$\sum a_n$ converges $\Leftrightarrow \sum b_n$ converges

② $\lim \frac{a_n}{b_n} = 0$, $\sum b_n$ conv $\Rightarrow \sum a_n$ conv.

③ $\lim \frac{a_n}{b_n} = +\infty$, $\sum b_n$ div $\Rightarrow \sum a_n$ div.

Ex 2 (a) $\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{n^3} \sim \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n+2}}{n^3}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{\sqrt{n+2}}{n} \overset{\text{LH}}{=} \lim \frac{\frac{1}{2\sqrt{n+2}}}{1} = \frac{1}{+\infty} = 0$$

$\sum \frac{1}{n^2}$ conv $(p=2>1$ p-series$)$

$\therefore$ LCT $\Rightarrow \sum \frac{5n+2}{n^3}$ conv.

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+10}} \sim \frac{1}{\sqrt{n}}$

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n+10}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+10}} =$$

$$\lim_{n\to\infty} \sqrt{\frac{n}{n+10}} =$$

$$\sqrt{\left(\lim_{n\to\infty} \frac{n}{n+1}\right)} = \sqrt{1} = 1 \neq 0$$

$\sum \frac{1}{\sqrt{n}}$ div $(p=\frac{1}{2}\leq 1$ p-series$)$

$\therefore$ LCT $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+10}}$

(c) $\sum_{n=3}^{\infty} \frac{12}{\sqrt[8]{n^9-17}} \sim \frac{1}{n^{9/8}}$

$$\lim_{n\to\infty} \frac{\frac{12}{\sqrt[8]{n^9-17}}}{\frac{1}{\sqrt[8]{n^9}}} = \lim_{n\to\infty} \frac{12\sqrt[8]{n^9}}{\sqrt[8]{n^9-17}}$$

$$= \lim_{n\to\infty} 12\sqrt[8]{\frac{n^9}{n^9-17}}$$

$$= 12\sqrt[8]{\lim \frac{n^9}{n^9-17}} = 12 \neq 0$$

$\sum \frac{1}{n^{9/8}}$ conv $\left(p = 9/8 > 1\right)$

$\therefore \sum \frac{12}{\sqrt[8]{n^9-17}}$ conv by LCT