3/30/ Calc2

Exam 2 Tomorrow

Review

HW 13 — type solution / HW outside

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Last time

9.2 Series : $\{a_k\}$

Defn $\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} S_n$

use sequences to define series

$S_n = \sum_{k=1}^{n} a_k$

partial sums

Telescoping series

Geometric series

$$\sum_{k=0}^{\infty} ar^k = \begin{cases} \frac{a}{1-r} & |r|<1 \\ \text{diverges} & |r|\geq 1 \end{cases}$$

$a + ar + ar^2 + ar^3 + \ldots$

# Combinations

$n^{\underline{th}}$ term test (divergence test)

$$\lim_{k\to\infty} a_k \neq 0 \Rightarrow \sum_{k=1}^{\infty} a_k \text{ diverges}$$

Ex) Apply $n^{\underline{th}}$ term test)

~~Ex)~~ (a) $\sum_{k=101}^{\infty} \left(1-\frac{100}{101}\right)^k =$

$$\lim_{k\to\infty} \left(1-\frac{100}{101}\right)^k = \lim_{k\to\infty} \left(\frac{1}{101}\right)^k = 0$$

Test fails;

But series geometric

$$\left(\frac{1}{101}\right)^{101} + \left(\frac{1}{101}\right)^{102} + \left(\frac{1}{101}\right)^{103} + \ldots$$

converges to $\dfrac{\left(\frac{1}{101}\right)^{101}}{\left(1-\frac{100}{101}\right)} = \left(\frac{1}{101}\right)^{100}$

(b) $\sum_{k=101}^{\infty}\left(1-\frac{100}{k}\right)^{101}$

$$\lim_{n\to\infty}\left(1-\frac{100}{n}\right)^{101}=1\neq 0$$

$\therefore$ series diverges.

(c) $\sum_{k=101}^{\infty}\left(1-\frac{100}{k}\right)^{k}$

$$\lim_{n\to\infty}\left(1-\frac{100}{n}\right)^{n}=L$$

$n \to \infty$

$1$

$$\ln L=\lim_{n\to\infty}\ln\left(1-\frac{100}{n}\right)^{n}=$$

$$\lim_{n\to\infty} n\ln\left(1-\frac{100}{n}\right)=$$

$$\lim_{n\to\infty}\frac{\ln\left(1-\frac{100}{n}\right)}{\left(\frac{1}{n}\right)}\overset{LH}{=}$$

$$\lim_{n\to\infty} \frac{\frac{1}{\left(1-\frac{100}{n}\right)} \cdot \cancel{\frac{100}{n^2}}}{-\cancel{\frac{1}{n^2}}}$$

$$\lim_{n\to\infty} \frac{-100}{\left(1-\frac{100}{n}\right)} = -100$$

$\frac{100}{n} \to 0$

So $L = e^{-100} = \frac{1}{e^{100}} \neq 0$

(note $3.72 \times 10^{-44}$)

<u>Ex2</u>: $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \ldots$

$\lim_{n\to\infty} \frac{1}{n^2} = 0$, <u>test fails</u>

Not telescoping

Not geometric

Conclusion?

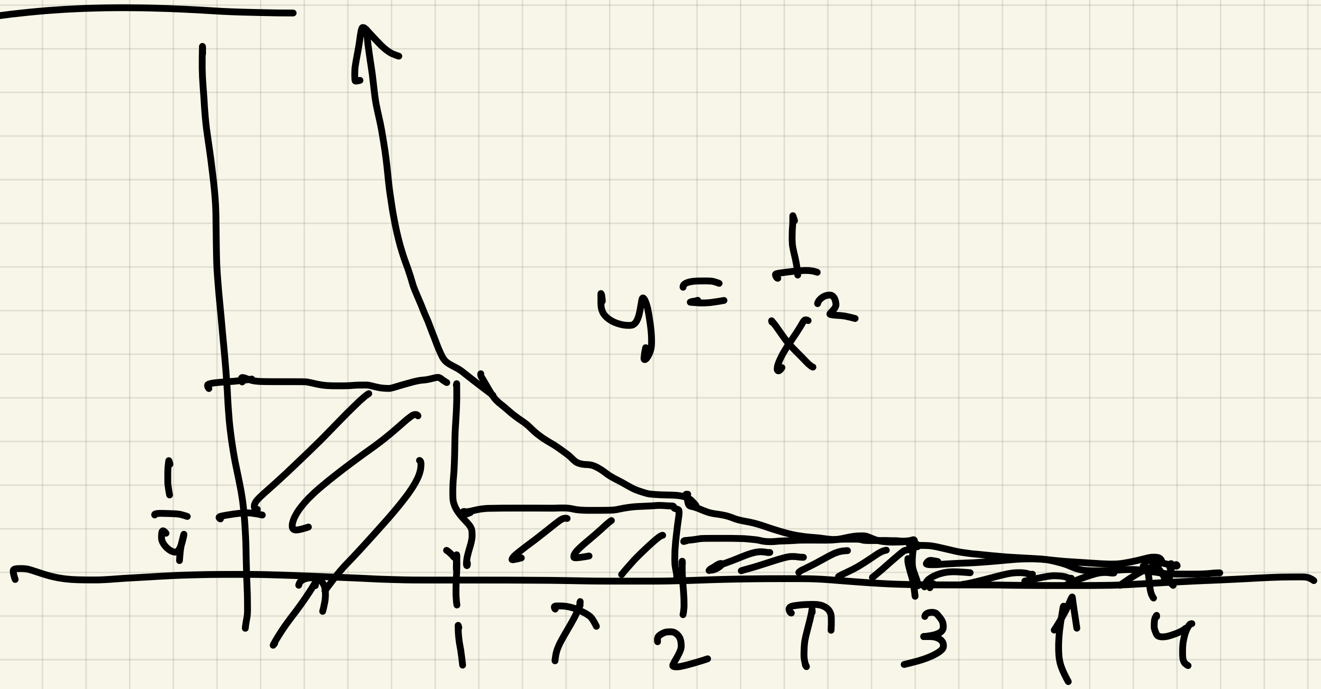
$s_n = 1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2}$

① $s_n$ bounded below 1

② $s_n$ monotonic (increasing)

So $\lim_{n\to\infty} s_n$ exists $\Leftrightarrow$ $\{s_n\}$ are bounded above.

Picture:



$1 \qquad \frac{1}{4} \qquad \frac{1}{9} \qquad \frac{1}{16}$

<u>Here</u> :

$$s_n = \sum \left( \begin{matrix} \text{area of} \\ \text{boxes} \end{matrix} \right) =$$

$$1 + \sum \left( \begin{matrix} \text{area of boxes} \\ \text{past first one} \end{matrix} \right)$$

$\leq 1 +$ Area under graph of $y = \frac{1}{x^2}$ from 1 to $\infty$

$$1 + \int_1^\infty \frac{1}{x^2} = 1 + \left( 1 \right) = 2$$

$p = 2$

So $s_n$ bounded above by $s_n \leq 2$

$\therefore$ series converges

(in fact, $\sum_{k=1}^{\infty} \frac{1}{k^2}$ to $\pi^2/6$)