3/31 Calc 2 Quiz 9

$\boxed{1}$ $\int \sin^3 x \cos^2 x \, dx$

$= \int (1-\cos^2 x)\cos^2 x \, \underline{\sin x} \, dx$

$\boxed{u = \cos x}$

$du = -\sin x \, dx$

$= -\int (1-u^2)u^2 \, du =$

$\int u^4 - u^2 \, du =$

$\frac{u^5}{5} - \frac{u^3}{3} + C =$

$\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C$

$\boxed{2}$ $\int \sec^4 x \tan^3 x \, dx =$

$$\int \sec^3 x \underbrace{(\sec^2 x - 1)}_{\tan^2 x} \sec x \tan x \, dx$$

$\cos^2 x + \sin^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$\div \cos^2 x$

$$\int u^3 (u^2 - 1) \, du$$

$$\int u^5 - u^3 \, du = \frac{u^6}{6} - \frac{u^4}{4} + C$$

$$= \frac{\sec^6 x}{6} - \frac{\sec^4 x}{4} + C$$

<u>Ex</u> $\displaystyle\int \frac{2}{x^2 - 1} \, dx$ $\qquad x = \sec\theta$

$dx = \sec\theta \tan\theta$

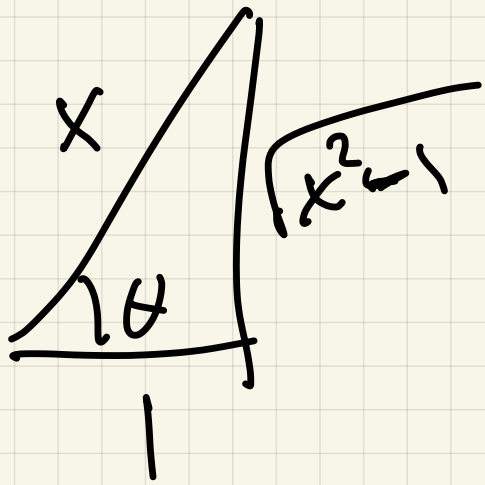
$$\int \frac{2 \sec\theta \tan\theta}{\tan^2\theta} \, d\theta =$$

$$\int \frac{2 \sec\theta}{\tan\theta} \, d\theta = \int \frac{2}{\sin\theta} \, d\theta =$$

$$\int 2\csc\theta \, d\theta =$$

$$-2\ln|\csc\theta + \cot\theta| + C$$

$$\|$$




$$-2\ln\left|\frac{x}{\sqrt{x^2-1}} + \frac{1}{\sqrt{x^2-1}}\right| + C$$

$$\|$$

$$-2\ln\left|\frac{x+1}{\sqrt{x^2-1}}\right| =$$

$$2\ln\left|\frac{\sqrt{x^2-1}}{x+1}\right| + C = \ln\left|\frac{x^2-1}{(x+1)^2}\right| + C$$

$$= \ln\left|\frac{(x-1)\cancel{(x+1)}}{(x+1)^{\cancel{2}}}\right| + C$$

$$= \ln\left|\frac{x-1}{x+1}\right| + C \quad \checkmark$$

<u>Better way:</u>

<u>observe</u>: $\displaystyle\int \frac{2}{x^2-1}\,dx = \int \frac{2}{(x-1)(x+1)}\,dx$

$$\int \frac{1}{x-1} - \frac{1}{x+1}\,dx$$

$$\ln|x-1| - \ln|x+1| + C$$

$$\ln\left|\frac{x-1}{x+1}\right| + C$$

## § 8.4 <u>Partial Fractions</u>

Undo fraction simplifying

<u>Ex1</u>: $\displaystyle\int \frac{4x+4}{x^2+2x-3}$ ←

$$(x+\underline{\underline{3}})(x-\underline{\underline{1}})$$

$$\int \frac{4x+4}{(x+3)(x-1)}\,dx$$

<u>Want</u>:

$$\frac{4x+4}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$

Solve for A & B:

mult by $(x+3)(x-1)$

$$4x+4 = \underline{A(x-1)} + B(x+3)$$

$x=1 \Rightarrow 8 = B(4) \Rightarrow B=2$

$x=-3 \qquad -8 = A(-4) \Rightarrow A=2$

$$\int \frac{4x+4}{(x+3)(x-1)}\,dx = \int \frac{2}{x+3} + \frac{2}{x-1}\,dx$$

$$= 2\ln|x+3| + 2\ln|x-1| + C$$

# Method of Partial Fractions

Wanted first $\displaystyle\int \frac{N(x)}{D(x)}\,dx$

polynomials

0. If $\deg N(x) \geq \deg D(x)$ long division to reduce to $\deg N(x) < \deg D(x)$

1. Factor $\underline{D(x)}$ into powers of linear / quadratic

$\boxed{(px+q)^m}$ $\boxed{(ax^2+bx+c)^n}$

$x^2+1$

2. <u>Form</u> of partial fractions:

- For each $(px+q)^m$, include

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \frac{A_3}{(px+q)^3} + \frac{A_m}{(px+q)^m}$$

- For each $(ax^2+bx+c)^n$, include

$$\frac{B_1x+C_1}{(ax^2+bx+c)} + \frac{B_2x+C_2}{(\quad)^2} + \cdots \frac{B_nx+C_n}{(\quad)^n}$$

3. Solve for $A_i / B_i / C_i$ and integrate:

Ex 3 $\displaystyle\int \frac{2x+1}{x^3+3x^2+2x} =$ $x(x^2+3x+2)$

$\parallel$

$x(x+1)(x+2)$

so $$\frac{2x+1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$$

Solve for $A/B/C$

multiply by $x(x+1)(x+2)$

$$2x+1 = A(x+1)(x+2) + Bx(x+2) + Cx(x+1)$$

$$x=0 \Rightarrow 1 = A(1)(2) + \cancel{B \cdot 0} + \cancel{C \cdot 0}$$

$$x=-1 \Rightarrow -1 = B(-1)(1)$$

$$x=-2 \Rightarrow -3 = 0 + 0 + C(-2)(-1)$$

$$C = -3/2,\ B = 1,\ A = 1/2$$

$\underline{\underline{\int_D}}$

$$\int \frac{2x+1}{x^3+3x^2+2x} = \int \frac{1/2}{x} + \frac{1}{x+1} - \frac{3/2}{x+2}\,dx$$

$$= \frac{1}{2}\ln|x| + \ln|x+1| - 3/2 \ln|x+2| + C$$

$$= \frac{1}{2}\ln\left|\frac{x(x+1)^2}{(x+2)^3}\right| + C$$

<u>Ex 4</u>: $\displaystyle\int \frac{x+4}{x^4-2x^3}$

$$x^4 - 2x^3 = x^3(x-2)^1$$

<u>Form</u>:

$$\frac{x+4}{x^3(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-2}$$

multiply by $x^3(x-2)$

$$\boxed{x+4 = Ax^2(x-2) + Bx(x-2) + C(x-2) + Dx^3}$$

$$\parallel$$

$$x+4 = \underbrace{(A+D)}_{0}x^3 + (-2A+B)x^2 + (-2B+C)x + (-2C)$$

$$\underline{0}x^3 + 0x^2 + 1x + 4$$

line up coeffs:

$$\begin{array}{l} x^3: \\ x^2 \\ x: \\ x^0 \end{array} \begin{cases} 0 = A+D \\ 0 = -2A+B \\ 1 = -2B+C \longrightarrow 1 = -2B-2 \quad B = -3/2 \\ 4 = -2C \longrightarrow C = -2 \end{cases}$$

$3 = -2B$

$0 = -2A + (-3/2)$ $\qquad -3/2 = B$

$3/2 = -2A$ $\qquad A = -3/4$

$D = 3/4$

So

$$\int \frac{x+4}{x^3(x-2)} = \int -\frac{3/4}{x} + \boxed{\frac{-3/2}{x^2}} - \frac{-2}{x^3} + \frac{3/4}{x-2}$$

$$= -3/4 \ln|x| + \frac{3/2}{x} + \frac{1}{x^2} + 3/4 \ln|x-2| + C$$

$$= 3/4 \ln\left|\frac{x-2}{x}\right| + \frac{3/2}{x} + \frac{1}{x^2} + C$$

Note $x = 0 \Rightarrow C = -2$

$x = 2 \Rightarrow D = 3/4$

Ex4 $\displaystyle\int \frac{6x^2+x-10}{x^3+5x} =$ $\quad x(\underline{\underline{x^2+5}})$

$x^2+5=0$ has no solutions $\longleftrightarrow$ cannot be factored

Form of fraction

$$\frac{6x^2+x-10}{x(x^2+5)} = \frac{A}{x} + \frac{Bx+C}{x^2+5}$$

$$\Downarrow$$

$$6x^2+1x-10 = A(x^2+5) + (Bx+C)x$$

$$\underline{6}x^2+\underline{x}\,\underline{-10} = (A+B)x^2+\underline{C}x+\underline{5}A$$

$A+B=6$

$C=1$

$5A=-10$

$A=-2$

$C=1$

$B=8$

$$\int \frac{-2}{x} + \frac{8x+1}{x^2+5}\,dx$$