3/23 | Calc2

Quiz 12

(1) $\lim_{x\to 0} \frac{\cos 4x - 1}{3x^2} \underset{LH}{\overset{0/0}{=}} \lim_{x\to 0} \frac{-4\sin 4x}{6x} \underset{LH}{\overset{0/0}{=}}$

$\lim_{x\to 0} \frac{-16\cos 4x}{6} = \frac{-16}{6}$

(2) $\lim_{x\to 0^+} (1+2x)^{5/x} = y$

$\ln y = \lim_{x\to 0^+} \ln(1+2x)^{5/x} =$

$\lim_{x\to 0^+} \frac{5\boxed{\ln(1+2x)}}{x} \underset{LH}{\overset{0/0}{=}}$

$\lim_{x\to 0^+} \frac{5\frac{2}{1+2x}}{1} = 10 = \ln y$

$\boxed{y = e^{10}}$

[ Exam 1 → T/R next week

[§ 9.1 → next week]

§ 9.1 Sequences

numbers 2 5 8 11 14 . . .

formulas $a_n = 3n-1,\ n \geq 1$

$\lim_{n\to\infty} a_n = L \ /\ a_n \to L$

$a_n$ converges to L

Calcl is applicable $\boxed{\text{sometimes}}$

$\boxed{\text{Ex}}$ $\lim_{n\to\infty} \frac{3n^2+2}{6n^2+1} = \lim_{x\to\infty} \frac{3x^2+2}{6x^2+1} =$

(a) n int x real

$\lim_{x\to\infty} \frac{3 + 2/x^2}{6 + 1/x^2} = \frac{1}{2}$

(b) $\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{5n} = \lim_{u\to 0^+}(1+2u)^{5/u}$

|| Quiz 3

$e^{10}$

trick: $u = \frac{1}{n}$

$n = \frac{1}{u}$

Not always:

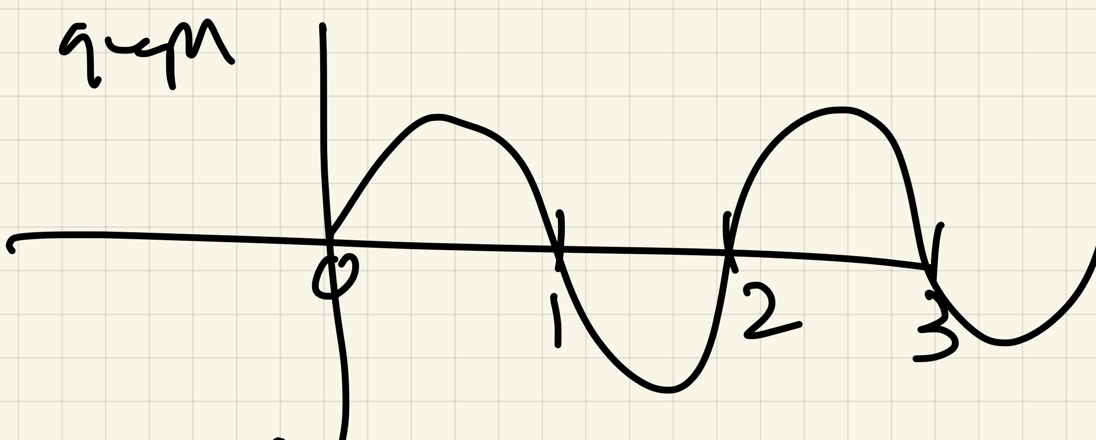
$$\boxed{\lim_{n\to\infty}\sin(n\pi) = 0}$$

|| Calc 2

$$\lim_{x\to\infty}\sin(x\pi) = \text{DNE}$$

$x$ real

graph




(b) $\lim_{n\to\infty}\cos(n\pi) = ??$

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $\cos(n\pi)$ | 1 | $-1$ | 1 | $-1$ | 1 | $-1$ |

even terms $1 \to 1$
odd terms $-1 \to -1$

So sequence <u>diverges</u>

<u>Note</u>: $\cos(n\pi) = (-1)^n$

(c) $\lim_{n \to \infty} \frac{\cos n\pi}{n} = a_n$

| $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $a_n$ | $-1$ | $1/2$ | $-1/3$ | $1/4$ | $-1/5$ |

Can see from table that

$$\lim \frac{\cos n\pi}{n} = 0$$

<u>odd terms</u> $\lim_{n\to\infty} \frac{\cos(2n+1)\pi}{2n+1}$

$= \lim \frac{-1}{2n+1} = 0$

## Tools for sequences

### Ⓐ <u>Sandwich theorem</u> :

If $\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} b_n$ and

$a_n \le c_n \le b_n$

then $\lim_{n\to\infty} c_n = L$

<u>Application</u> : $c_n = \frac{\cos n\pi}{n}$

$$\underset{\underset{a_n}{\shortparallel}}{\frac{-1}{n}} \le \frac{\cos n\pi}{n} \le \underset{\underset{b_n}{\shortparallel}}{\frac{1}{n}}$$

$\lim \frac{-1}{n} = 0 = \lim \frac{1}{n}$

$$\therefore \lim_{n\to\infty} \frac{\cos n\pi}{n} = 0$$

(B) $\lim_{n\to\infty} a_n = 0 \iff \lim_{n\to\infty} |a_n| = 0$

Application $\left|\frac{\cos n\pi}{n}\right| = \frac{1}{n}$

$$\lim \frac{1}{n} = 0 \Rightarrow \lim \frac{\cos n\pi}{n} = 0$$

(C) Combinations

If $\lim a_n = L$
$\lim b_n = M$

$$\Rightarrow \quad \lim (a_n + b_n) = L + M$$
$$\lim (a_n - b_n) = L - M$$
$$\lim (a_n \cdot b_n) = L \cdot M$$
$$\lim \frac{a_n}{b_n} = \frac{L}{M}, \quad M \neq 0$$

Ex $\lim \left(\frac{1}{e}\right)^n + \frac{n^2-1}{6-3n^2} + \arctan n$

$$0 + -\tfrac{1}{3} + \tfrac{\pi}{2}$$

$$= \tfrac{\pi}{2} - \tfrac{1}{3} = \frac{3\pi - 2}{6}$$

More vocabulary:

(1) A sequence $\{a_n\}$ is monotonic if

a) $a_1 \le a_2 \le a_3 \le \ldots$

OR

b) $a_1 \geq a_2 \geq a_3 \geq a_4 \geq \ldots$

(2) $\{a_n\}$ is bounded above (resp. below) if there is a number $M$:

$a_n \le M$ for all $n$.

(resp $(a_n \geq M)$)

(3) Connection:

If $\{a_n\}$ is monotonic,

then $a_n$ converges

$$\Updownarrow$$

$a_n$ bounded