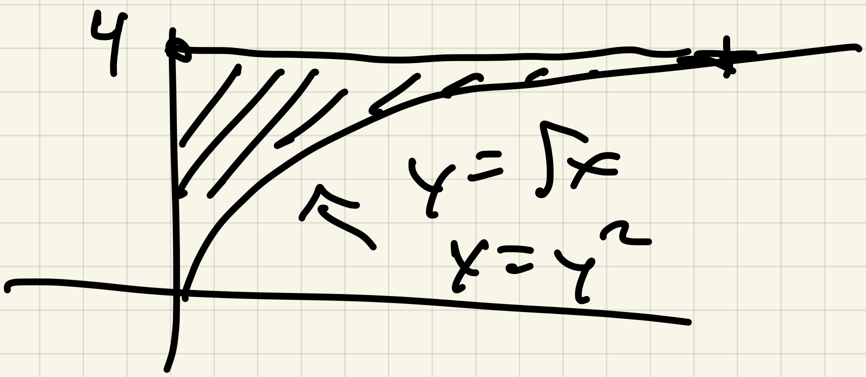
1/29 | Calc 2 Quiz 2



$4=\sqrt{x}$
$16=x$

$\boxed{1.}$ $A = \int_0^{16} 4-\sqrt{x}\,dx$

$= 4x - \frac{2}{3}x^{3/2}\Big|_0^{16} =$

$64 - \frac{2}{3}(64) = \frac{64}{3}$

$\boxed{2}$ $A = \int_0^4 y^2 - 0\,dy =$ // ✓

$y=\sqrt{x}$
$y^2=x$

$\frac{y^3}{3}\Big|_0^4 = \frac{64}{3}$

Last time $V = \int_a^b A(x)\,dx$

Disk $V = \int_a^b \pi (R(x)^2)\,dx$

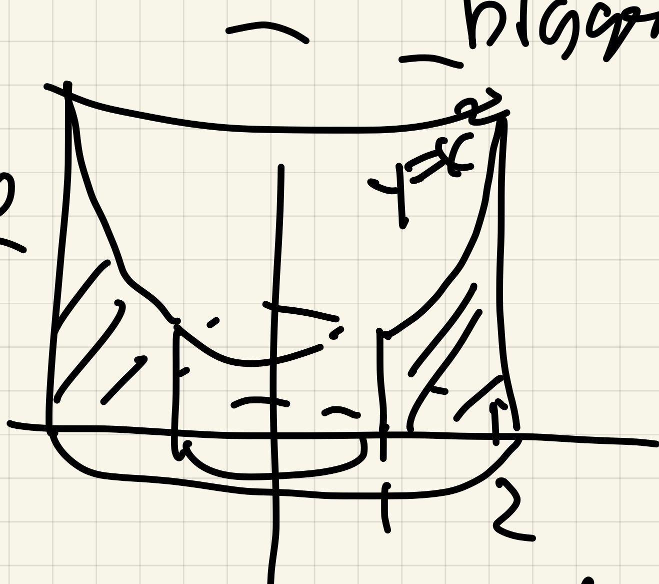
$R(x)$ = radius of disk

washer $\quad V = \int_a^b \pi \left(R(x)^2 - r(x)^2 \right) dx$

bigger rad

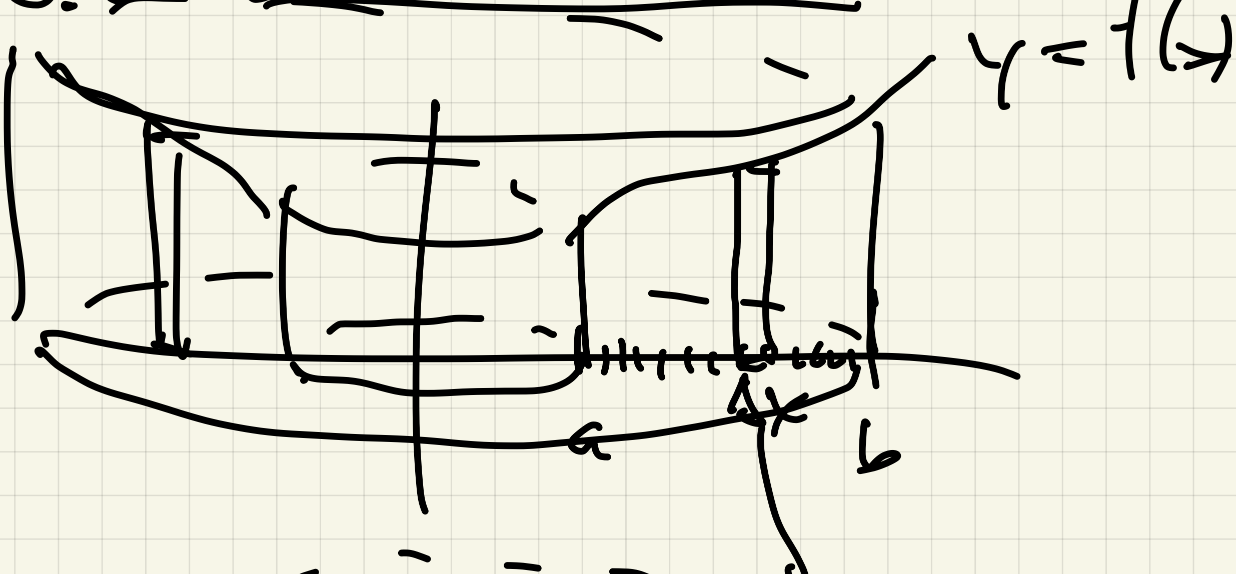
smaller radius

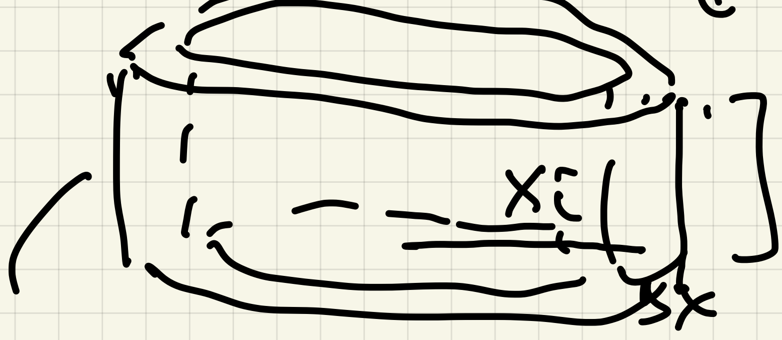
Ex 0



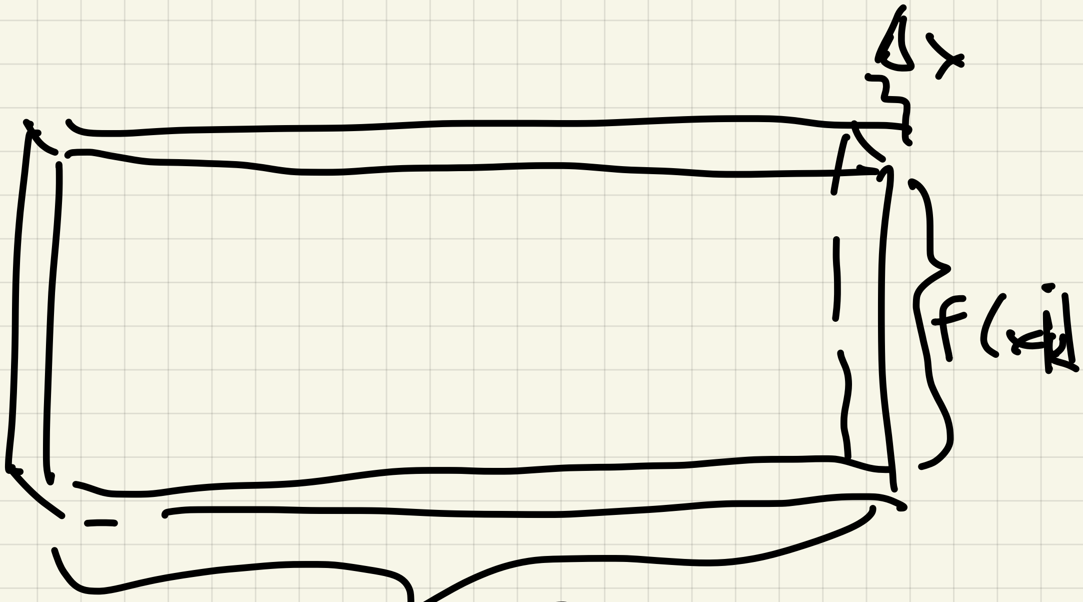
$V = \frac{15\pi}{2}$

§6.2 Shell method:





flatten



length = $2\pi x_k$

Volume of k^{th} shell is

$$V_k = \underbrace{2\pi x_i}_{l}\ \underbrace{f(x_i)}_{ht}\ \underbrace{\Delta x}_{w}$$

so $V \approx \sum_{k=1}^{n} 2\pi x_i \, f(x_i)\, \Delta x$

$\Downarrow$

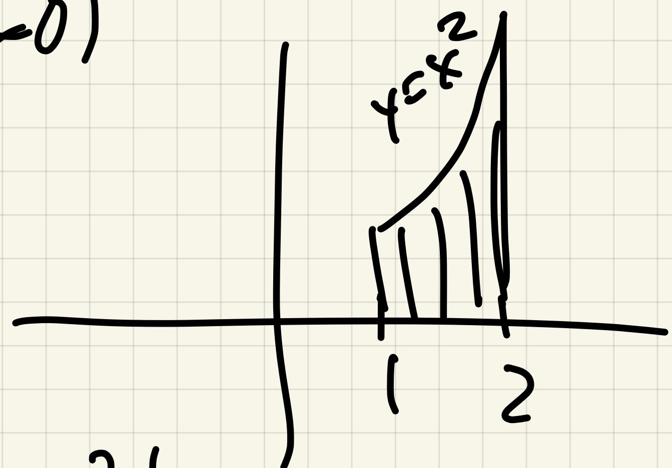
Exact volume = $V = \lim_{n\to\infty}\left(\qquad \right) =$

$$\boxed{\int_a^b 2\pi x \, f(x)\, dx}$$

$\|$

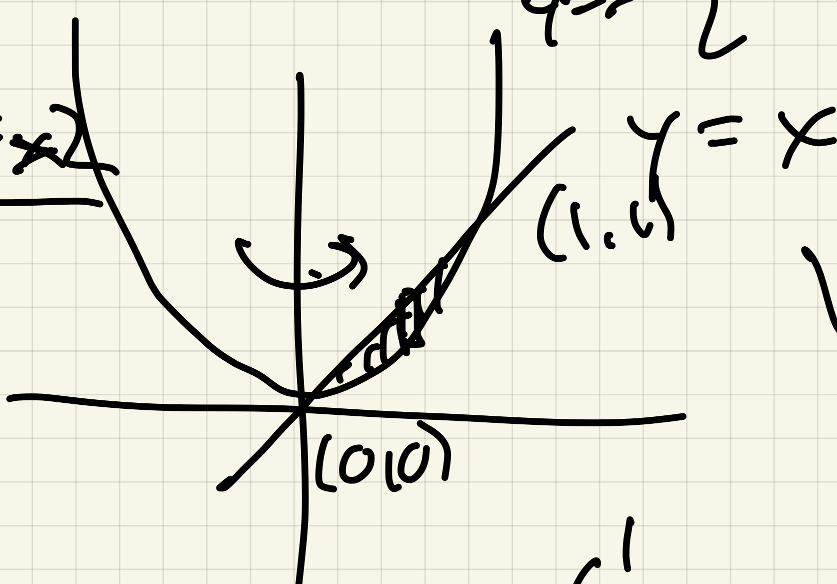
$$\boxed{\int_a^b 2\pi \begin{pmatrix}\text{radius}\\ \text{of}\\ \text{shell}\end{pmatrix}\cdot\begin{pmatrix}\text{ht of}\\ \text{shell}\end{pmatrix} dx}$$

Ex 1 (Ex 0)



$$\int_1^2 2\pi \underbrace{x}_{\text{rad}}\, \underbrace{x\,x^2}_{x^3}\, dx = 2\pi \frac{x^4}{4}\Big|_1^2 = \frac{\pi}{2}(2^4-1) = \frac{15\pi}{2}$$

Ex2



Volume of revolution

$$\int_0^1 2\pi x\cdot(x-x^2)\,dx$$

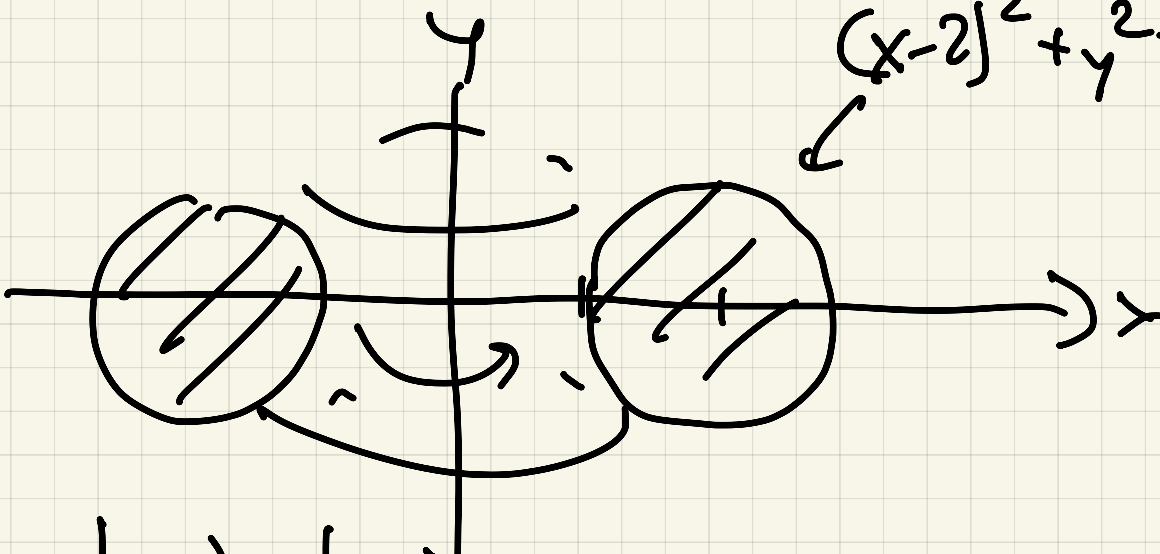
$$2\pi \int_0^1 x^2 - x^3 \, dx =$$

$$2\pi \left(\frac{x^3}{3} - \frac{x^4}{4} \right) \Big|_0^1 =$$

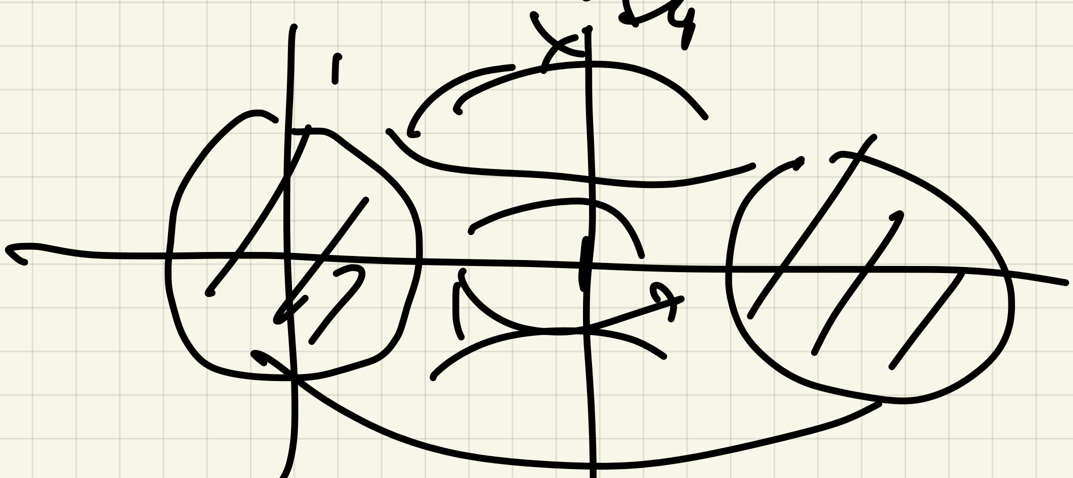
$$2\pi \left(\frac{1}{3} - \frac{1}{4} \right) = 2\pi \left(\frac{1}{12} \right) = \frac{\pi}{6}$$

<u>Ex 3</u>

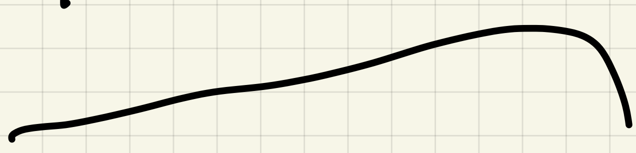
$(x-2)^2 + y^2 = 1$

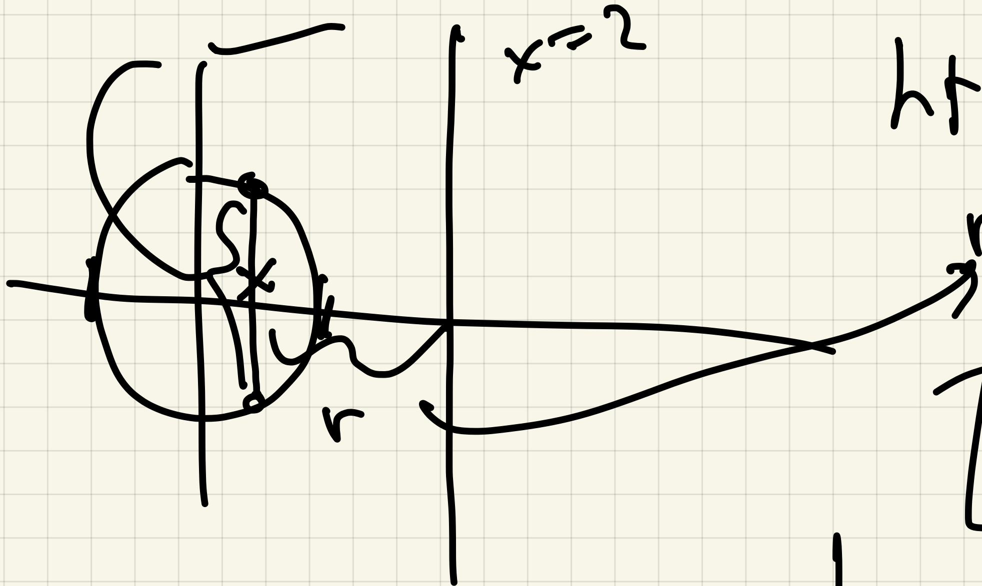


<u>Instead</u>: $x^2 + y^2 = 1$



$x = 2$



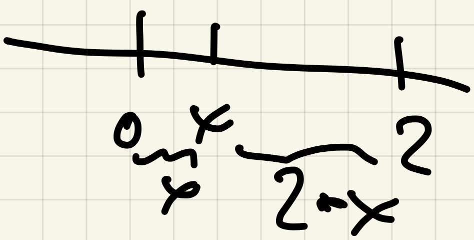


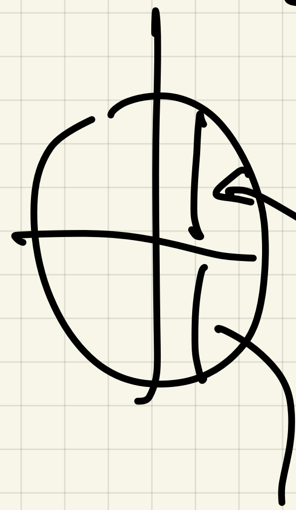
ht of shell

r = radius

||

$$\boxed{2-x}$$





$x^2+y^2=1$

$y = \pm\sqrt{1-x^2}$

$y = \sqrt{1-x^2}$

$y = -\sqrt{1-x^2}$

ht of shell is

$$\boxed{2\sqrt{1-x^2}}$$

Shell method $= \displaystyle\int_{-1}^{1} 2\pi(2-x)\,2\sqrt{1-x^2}\,dx$

$$4\pi\int_{-1}^{1}(2-x)\underbrace{\sqrt{1-x^2}}_{=}\,dx =$$

0

$$4\pi\int_{-1}^{1} 2\sqrt{1-x^2} \;-\; \cancel{4\pi\int_{-1}^{1} x\sqrt{1-x^2}\,dx}$$

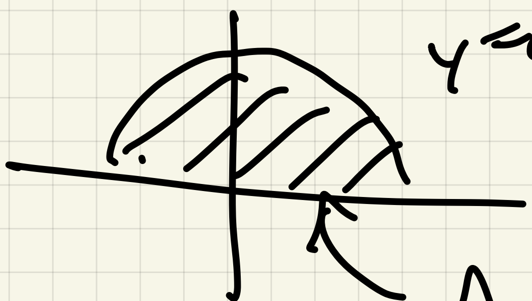
$$\|$$

$$\underbrace{8\pi\int_{-1}^{1}\sqrt{1-x^2}\,dx}$$

$u = 1-x^2$

$du = -2x\,dx$

$$\int_{u=0}^{u=0} \underline{\qquad\qquad} = 0$$

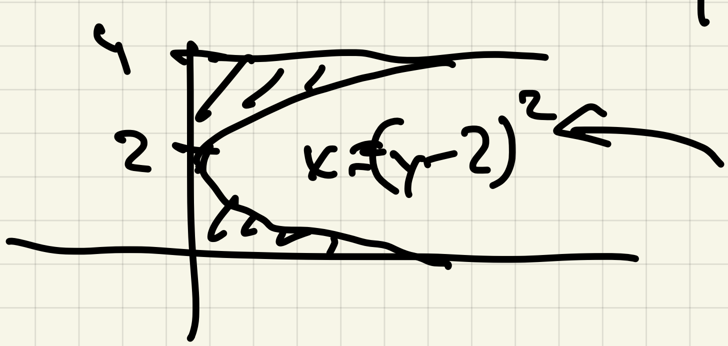


$y=\sqrt{1-x^2}$

Area $= \pi/2$

$$V = 8\pi\left(\frac{\pi}{2}\right) = 4\pi^2 \checkmark$$

__Ex 4__



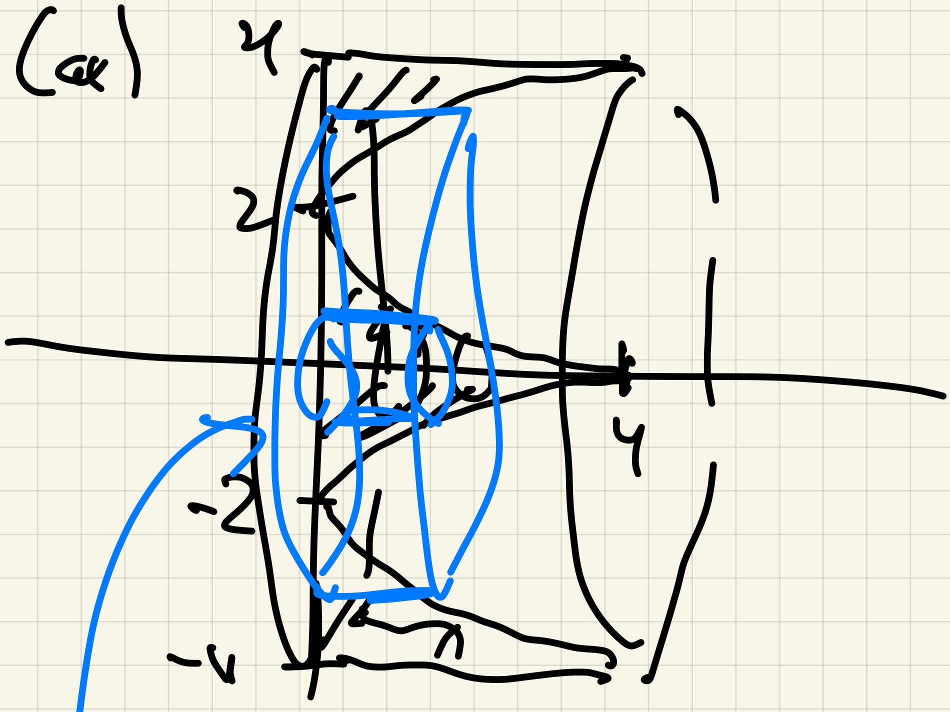
Find volume when region is revolved about

(a) x-axis

(b) $y=-2$

(c) y-axis

(a)



"radius of shell" y

"ht" of shell is $x = (y-2)^2$

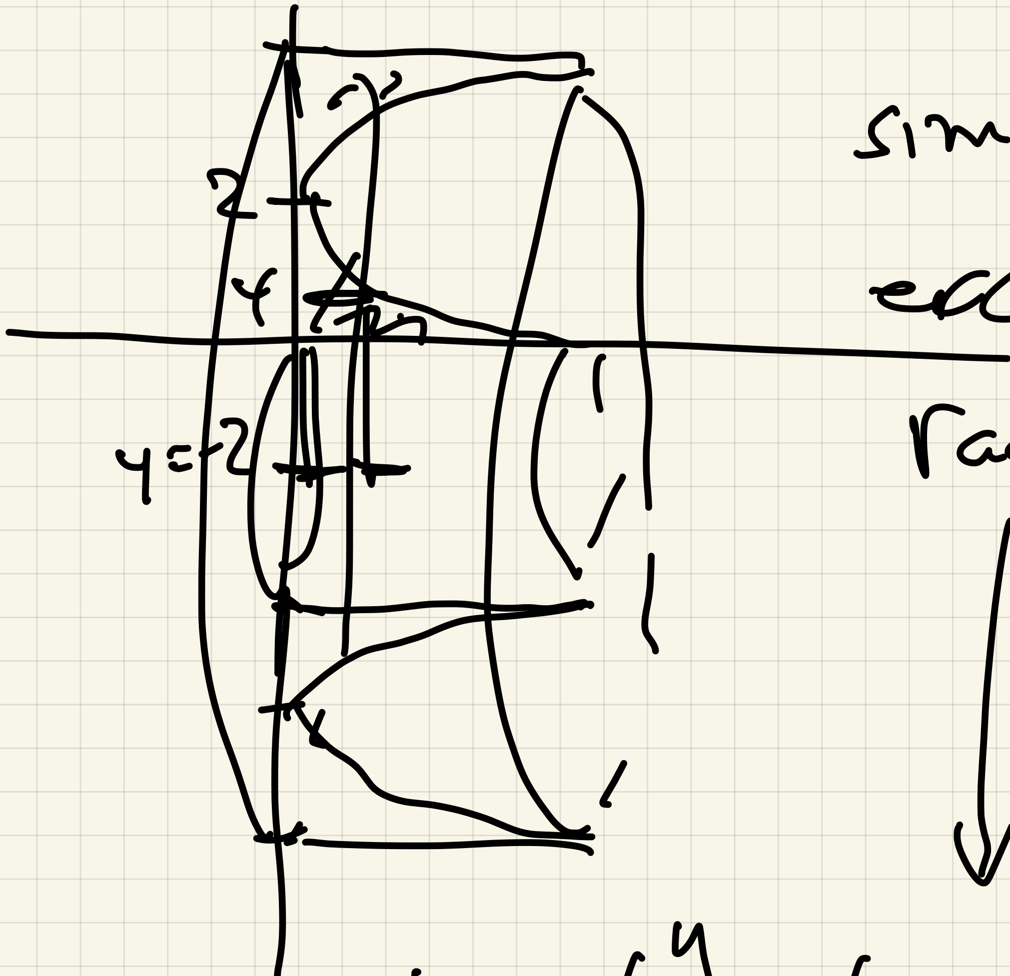
$$V = \int_{y=0}^{y=4} 2\pi y (y-2)^2 \, dy$$

$$y(y^2-4y+4)$$

$$\int_0^4 2\pi y (y-2)^2 \, dy =$$

$$2\pi \int_0^4 y^3 - 4y^2 + 4y \, dy = \frac{64\pi}{3}$$

(b)



similar to (a),

except:

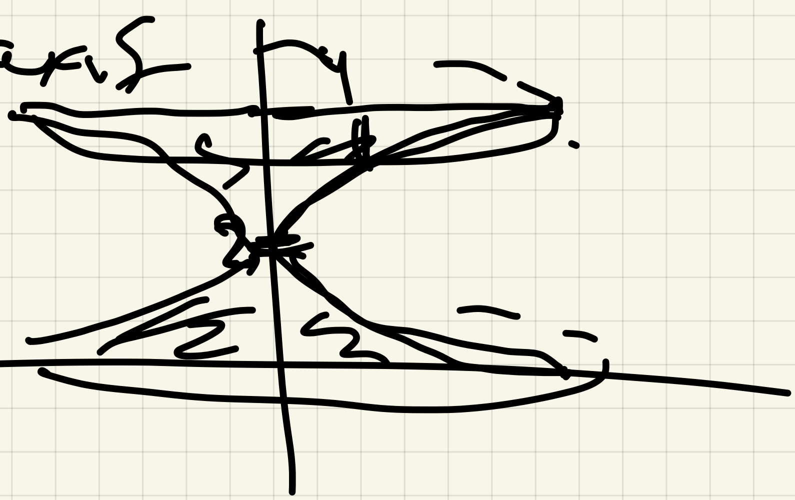
radius = y+2

$$V=\int_0^4 2\pi(y+2)(y-2)^2\,dy$$

4

$$\frac{128\pi}{3}$$

(c) y-axis



Shells:

$x = (y-2)^2 \Rightarrow \pm\sqrt{x} = y-2$

$y = 2 \pm \sqrt{x}$

ht of shells:

top $4 - (2+\sqrt{x})$

bottom $(2-\sqrt{x}) - 0$

$$V = \int_0^4 2\pi x\left(2-\sqrt{x}\right)dx \quad \text{top}$$

$$+$$

$$\int_0^4 2\pi x\left(2-\sqrt{x}\right)dx \quad \text{bottom}$$

BUT

disks much better:

$$\int_0^4 \pi\left((y-2)^2\right)^2 dy =$$

$$\int_0^4 \pi (y-2)^4 \, dy \qquad u = y-2, \quad du = dy$$

$$\int_{-2}^{2} \pi u^4 \, du = \pi \frac{u^5}{5}\Bigg|_{-2}^{2} =$$

$$\frac{32\pi}{5} - \left(-\frac{32\pi}{5}\right) = \frac{64\pi}{5}$$

Question: When are shell/washers better

	x in terms of y	y in terms of x (picture)
revolving about horizontal axis (x-axis)	shells	disks/washers
vertical (y-axis)	disk/washer	shells