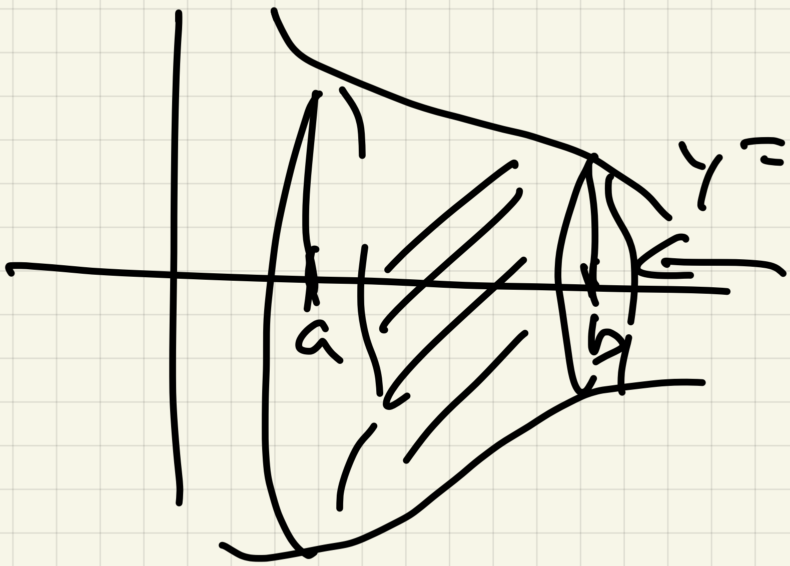
1/22/ Calc2

Volume $= \int_a^b A(x)\,dx$

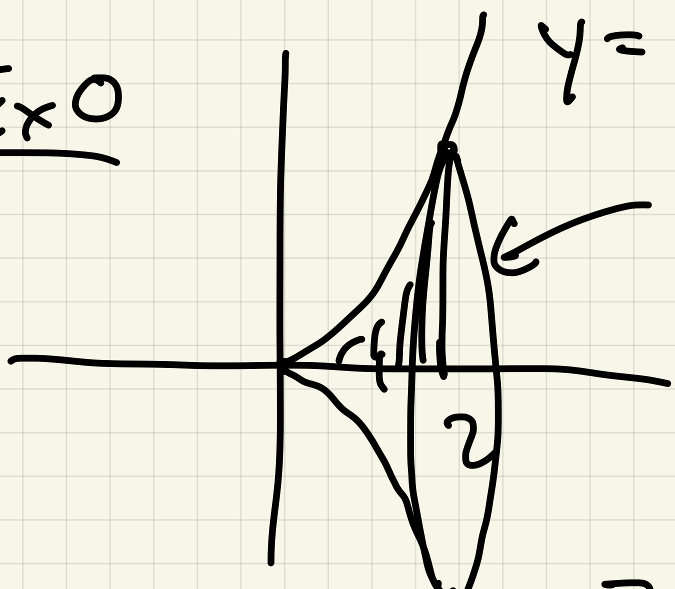


$y = f(x)$

Volume is

$$\int_a^b \pi (f(x))^2\,dx$$

Ex 0



$y = x^3$

Volume $= \boxed{\int_0^2 \pi (x^3)^2\,dx}$

$$= \int \pi x^6\,dx =$$

$$\pi \frac{x^7}{7}\Big|_0^2 = \frac{128}{7}\pi$$

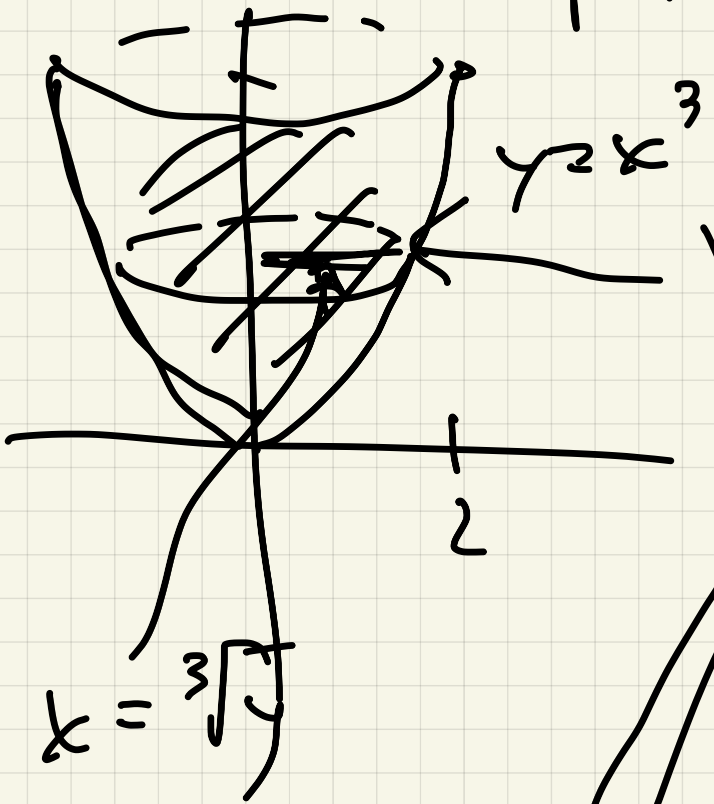
112

18.3π

Variations:

Ex1 $y=x^3$, $0 \le x \le 2$

Revolve about y-axis, compute volume



$$V = \int_0^8 \pi \overbrace{\left(\sqrt[3]{y}\right)^2}^{R^2} \, dy$$

$R=$

$y = x^3 \Rightarrow$

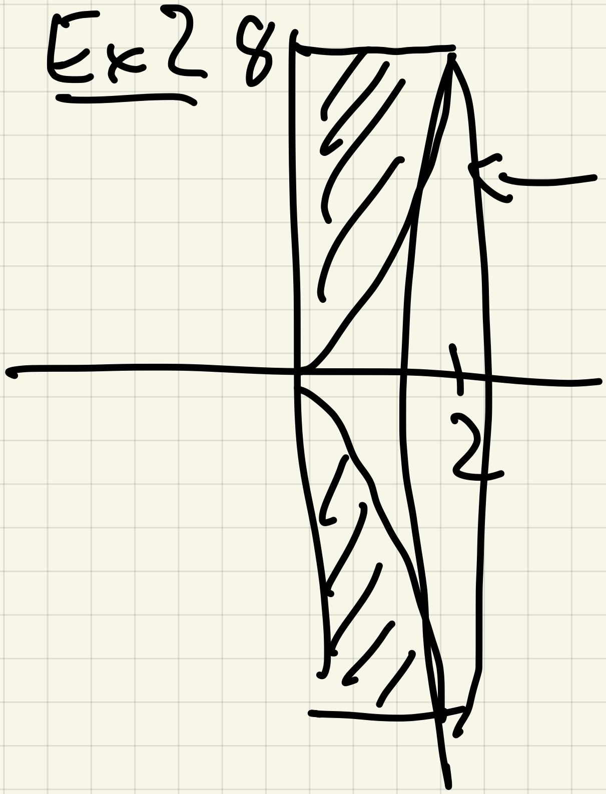
$x = \sqrt[3]{y}$

$$\int_0^8 \pi \left(y^{1/3}\right)^2 dy = \int_0^8 \pi y^{2/3} \, dy =$$

$$\pi \frac{3}{5} y^{5/3} \Big|_0^8 = \pi \frac{3}{5} 8^{5/3} =$$

$$\pi \frac{3}{5} \left(8^{1/3}\right)^5 = \pi \, 3/5 \, 2^5 = \pi \, 3/5 \cdot 32 = \frac{96}{5}\pi = 19.2\pi$$

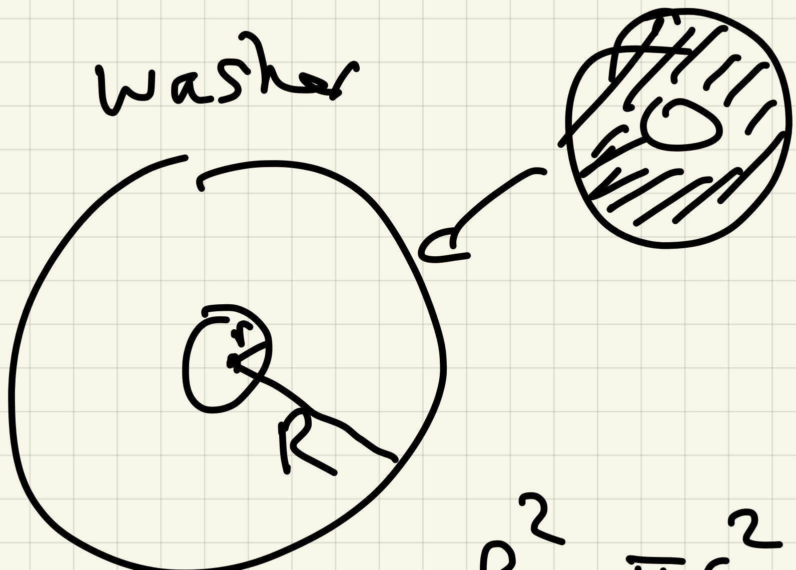
Ex 2



cross-sections

not circle

washer



Area = $\pi R^2 - \pi r^2$

so

$$V = \int_0^2 \pi\, 8^2 - \pi (x^3)^2 \, dx =$$

$$\int_0^2 64\pi \, dx - \underbrace{\int_0^2 \pi (x^3)^2 \, dx}_{\frac{128\pi}{7}}$$

$$64\pi x \Big|_0^2$$

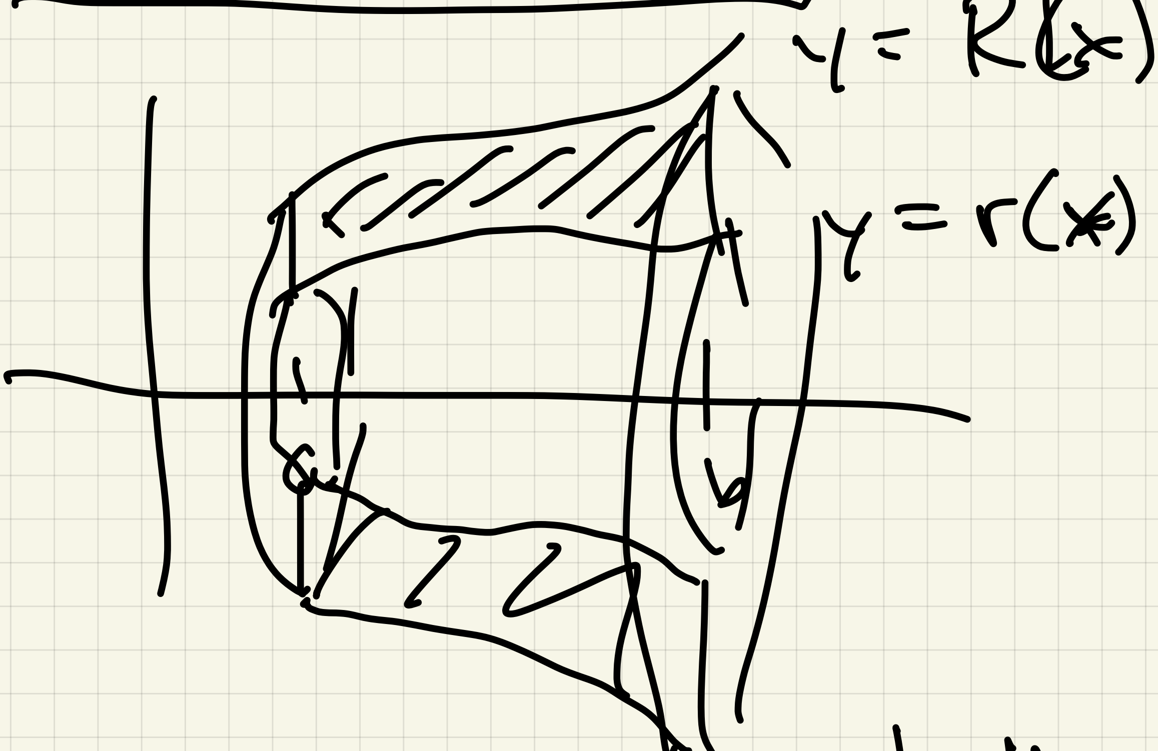
$\frac{128\pi}{7}$ Ex 0

$$128\pi - \frac{128\pi}{7} =$$

$$128\pi\left(1 - \frac{1}{7}\right) = 128\pi\left(\frac{6}{7}\right) =$$

$$\frac{768\pi}{7}$$

Washer method:



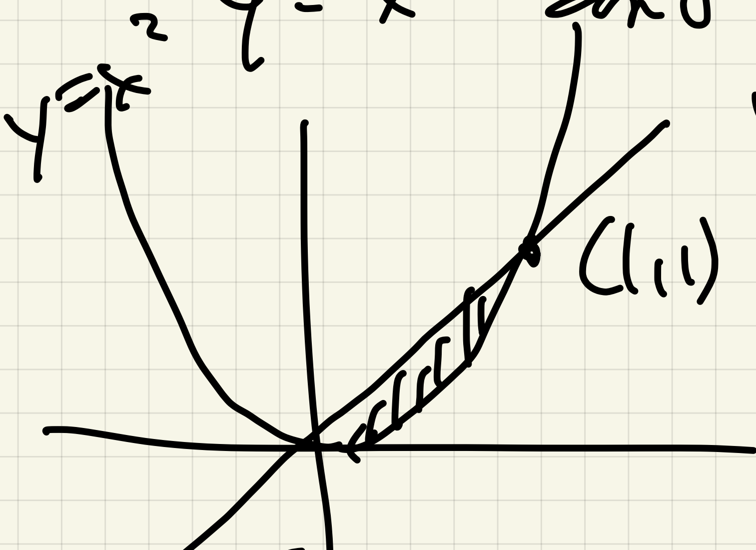
Volume of revolution is

$$\int_a^b \pi\left(R(x)^2 - r(x)^2\right)dx$$

Ex 3 Let S be shaded region between curves

$y = x$ and $y = x^2$ $\quad x = \sqrt{y}$

$y = x$ $\quad x = y$



Find volume ~~there~~ when S is revolved about:

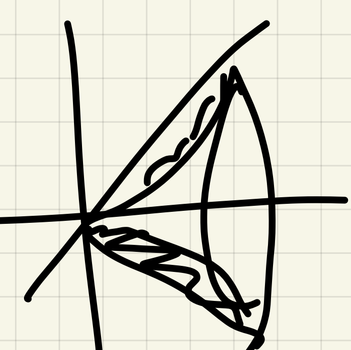
(a) x-axis

(b) y-axis

(c) line $y = 3$

(d) line $x = -1$ $\quad x^4$

(a) $\displaystyle\int_0^1 \pi \underbrace{x^2}_{R} - \pi \underbrace{(x^2)^2}_{r}\,dx$

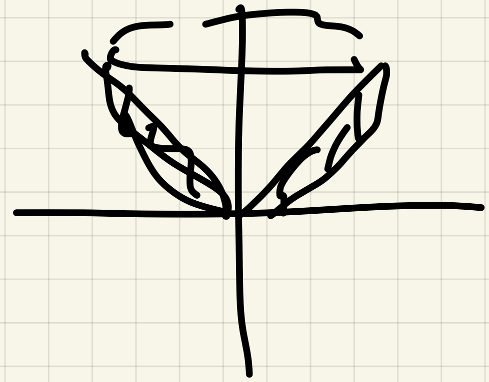


$$= \pi \left. \frac{x^3}{3} - \frac{x^5}{5} \right|_0^1 =$$

$$\pi\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\pi}{15}$$

(b) y=

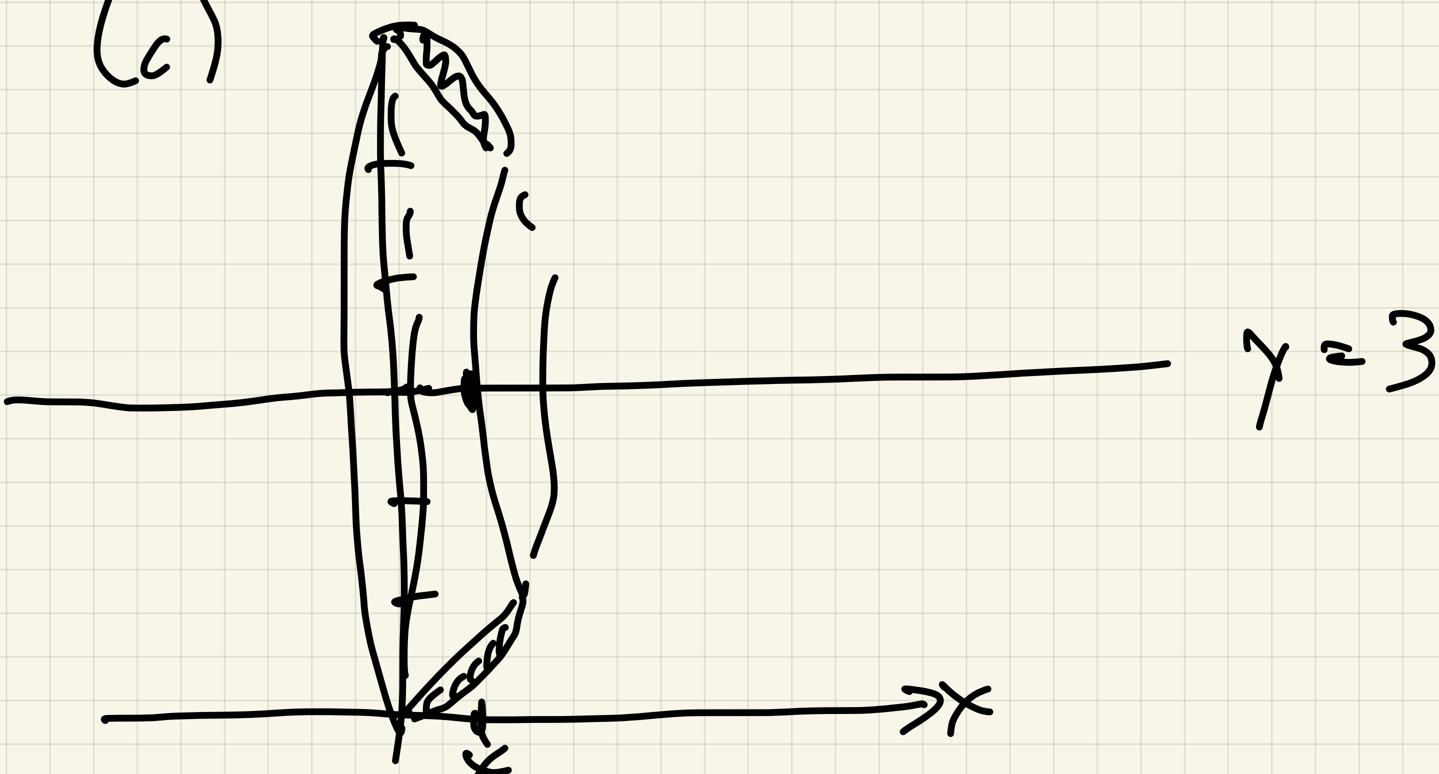
$$\int_0^1 \pi(\sqrt{y})^2 - \pi(y)^2\,dy$$



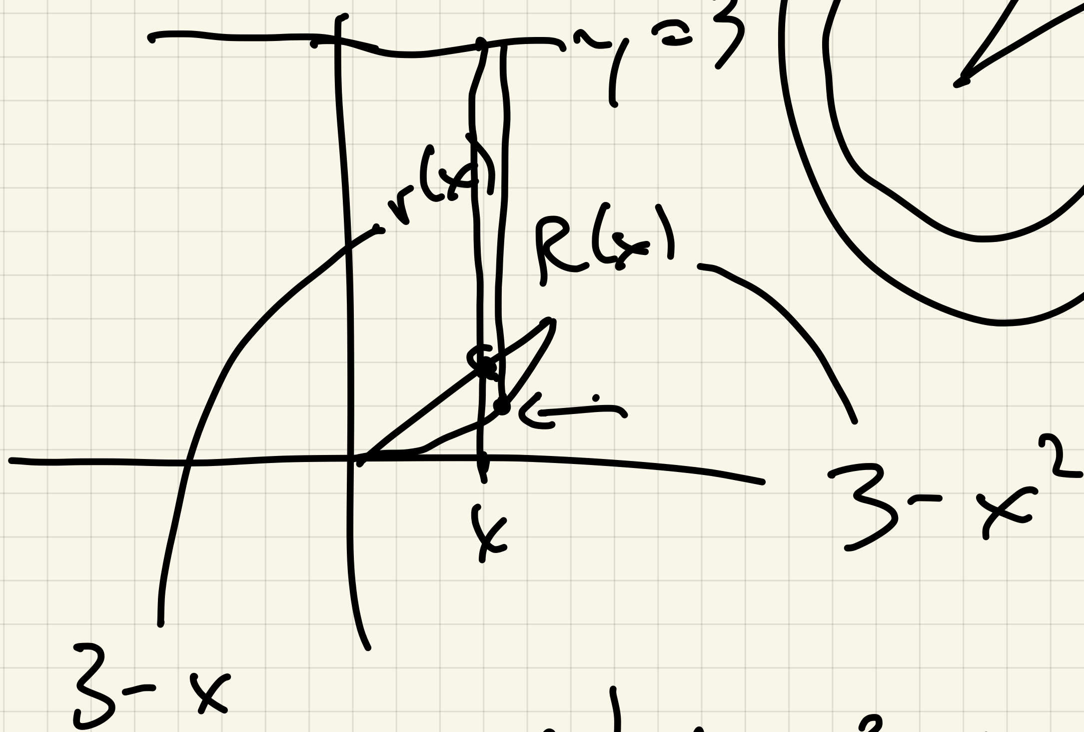
$$= \int_0^1 \pi y - \pi y^2\,dy =$$

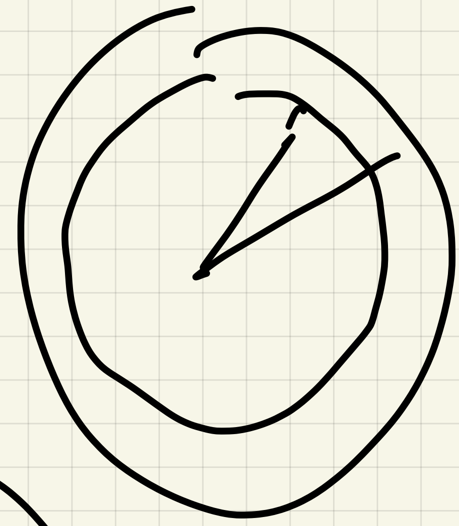
$$\pi\left(\frac{y^2}{2} - \frac{y^3}{3}\right)\Big|_0^1 = \frac{\pi}{6}$$

(c)



cross section





So Volume $= \int_0^1 \pi\left((3-x^2)^2-(3-x)^2\right)dx$

$$= \pi\int_0^1 (\cancel{9}-6x^2+x^4)-(\cancel{9}-6x+x^2)\,dx$$

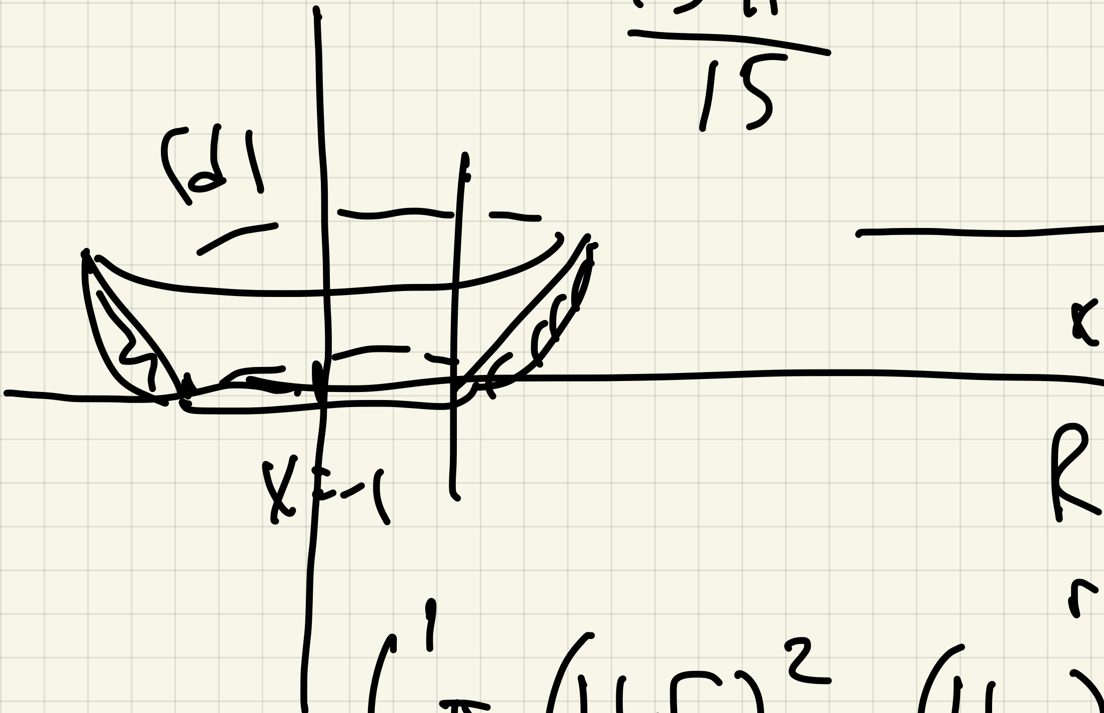
$$\pi\int_0^1 -7x^2+6x+x^4\,dx$$

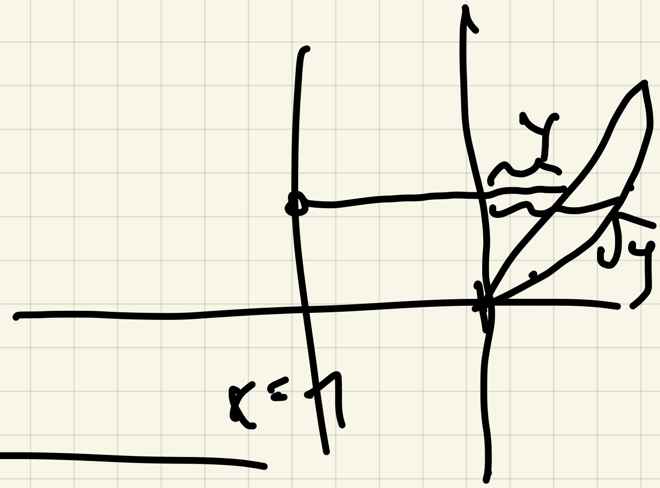
$$\pi\left(-\frac{7}{3}x^3+3x^2+\frac{x^5}{5}\right)\Big|_0^1 =$$

-35 +45 +3

$$\pi\left(-\frac{7}{3}+3+\frac{1}{5}\right) =$$

$$\frac{13\pi}{15}$$





$R = 1+\sqrt{y}$

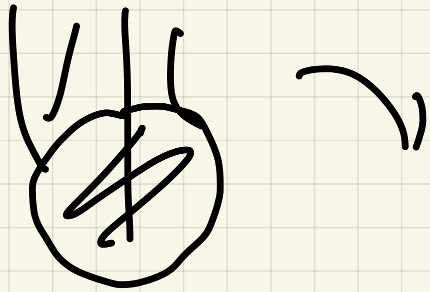
$r = 1+y$

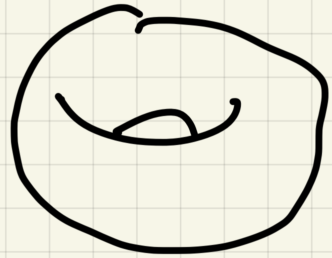
$$\int_0^1 \pi\left(1+\sqrt{y}\right)^2 - \left(1+y\right)^2\,dy =$$

$$\pi\int_0^1 1+2\sqrt{y}+y-1-2y-y^2\,dy = \frac{\pi}{2}$$

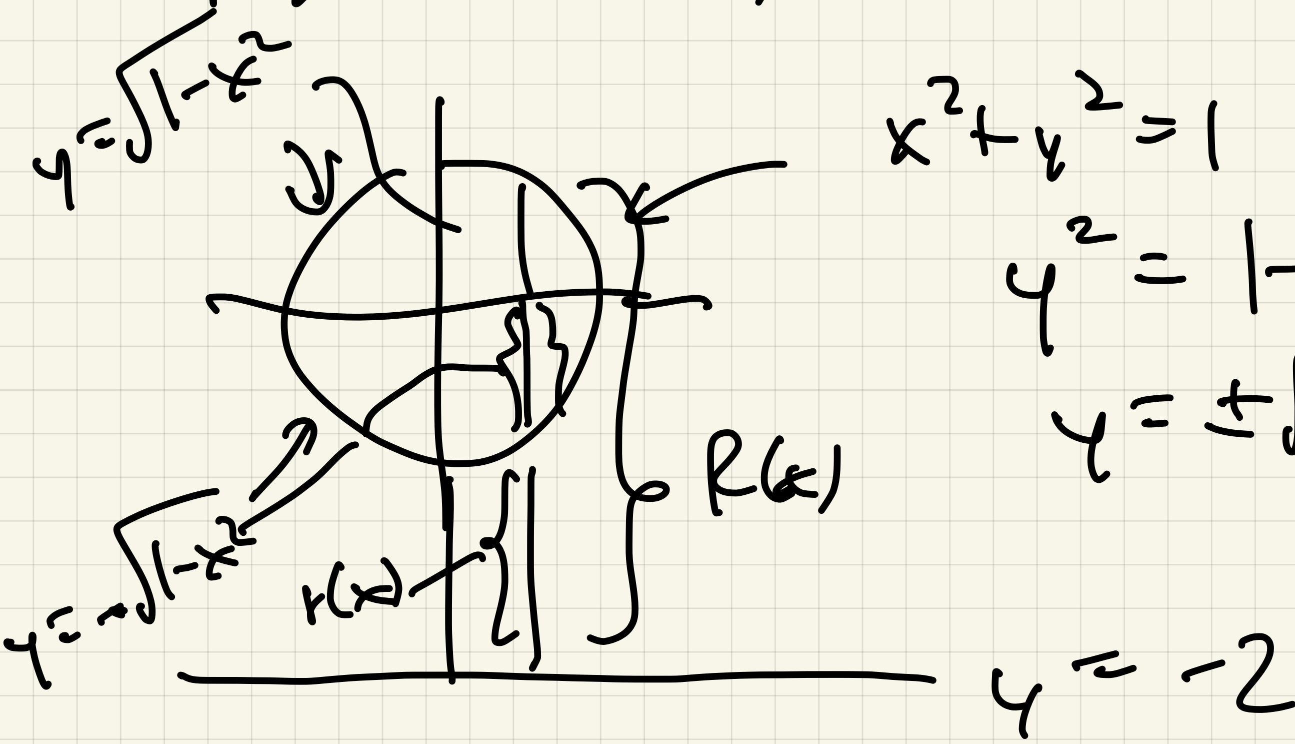
Ex4 Find volume of the disk $x^2+y^2 \le 1$ revolved about line $y=-2$

$y=-2$





$R(x) = ?$ $\quad r(x) = ?$



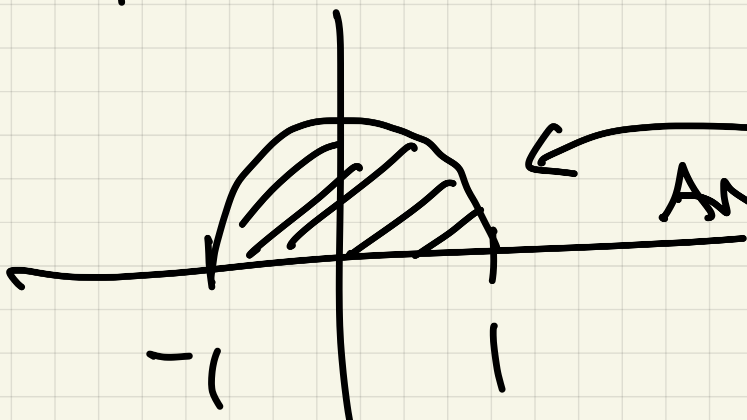
$$x^2 + y^2 = 1$$
$$y^2 = 1 - x^2$$
$$y = \pm\sqrt{1-x^2}$$

$$R(x) = 2 + \sqrt{1-x^2}$$
$$r(x) = 2 - \sqrt{1-x^2}$$

$$V = \int_{-1}^{1} \pi\left(\left(2+\sqrt{1-x^2}\right)^2 - \left(2-\sqrt{1-x^2}\right)^2\right)$$

$$\pi\int_{-1}^{1} \left(\cancel{4} + 4\sqrt{1-x^2} + \cancel{(1-x^2)}\right) - \left(\cancel{4} - 4\sqrt{1-x^2} + \cancel{(1-x^2)}\right)$$

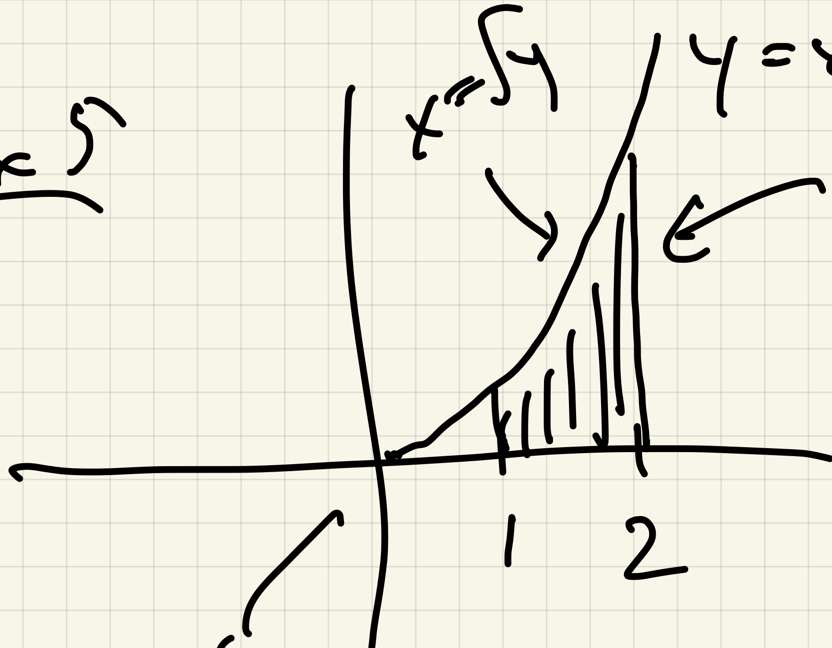
$$\pi \int_{-1}^{1} 8\sqrt{1-x^2} = 8\pi \underbrace{\int_{-1}^{1} \sqrt{1-x^2}\,dx}$$



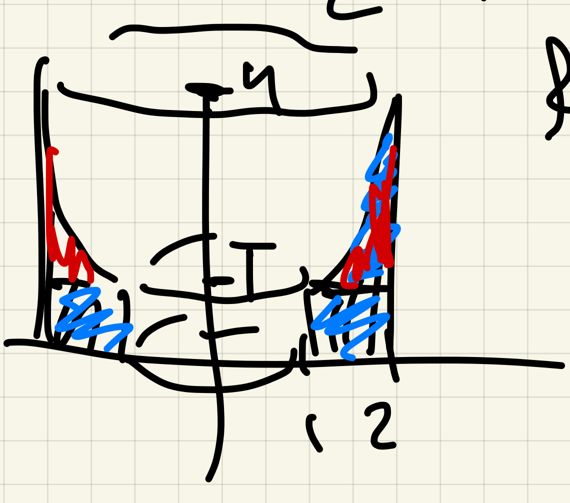
$$\text{Area} = \frac{\pi \cdot 1^2}{2}$$

So $V = 8\pi\left(\frac{\pi}{2}\right) = 4\pi^2$

Ex 5



Revolve about y-axis



$\cancel{R(y)}$ Out radius = 2

Inner radius:

$$r(y) = \begin{cases} \sqrt{y} & y > 1 \\ 1 & y \le 1 \end{cases}$$

Bottom $\int_0^1 \pi(2^2-1^2)\,dy =$

$3\pi y\Big|_0^1 = 3\pi$

Top $\int_1^4 \pi\left(2^2-(\sqrt{y})^2\right)dy =$

$\pi\int_1^4 4-y\,dy = \pi\left(4y-\frac{y^2}{2}\right)\Big|_1^4 =$

$\pi\left(\left(8\right)-\left(\frac{7}{2}\right)\right) =$

$\pi\left(\frac{16-7}{2}\right) = \frac{9\pi}{2}$

Total volume $\frac{9\pi}{2}+3\pi = \frac{15\pi}{2}$