1/20/ Calc2 Quiz1

(a) $\int 4x^6 - \frac{8}{x^4} + \frac{7}{x}\,dx$ $\quad -8x^{-4}$

$\frac{4}{7}x^7 \underbrace{- 8\,\frac{x^{-3}}{-3}} + 7\ln|x| + C$

$+\frac{8}{3x^3}$

(b) $\int 5e^x - \frac{1}{x^2+9} - 2\cos x\,dx$

$5e^x - \frac{1}{3}\arctan\frac{x}{3} - 2\sin x + C$

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}$

$\int \frac{dx}{1+x^2} = \arctan x + C$

$\int \frac{1}{x^2+9} = \int \frac{1/9}{\left(\frac{x}{3}\right)^2+1}$ $\quad u = \frac{x}{3}$, $du = \frac{1}{3}dx$

$$\int \frac{\frac{1}{9}\cdot 3\, du}{u^2+1} = \frac{1}{3}\arctan u$$

$3du = dx$

$u = \frac{x}{3}$

$\boxed{\#7}$

$$\int_0^4 6\sqrt{2x+1}\,dx$$

$u=9$

$u=1$

$\boxed{u = 2x+1}$

$du = 2\,dx$

$\frac{1}{2}du = dx$

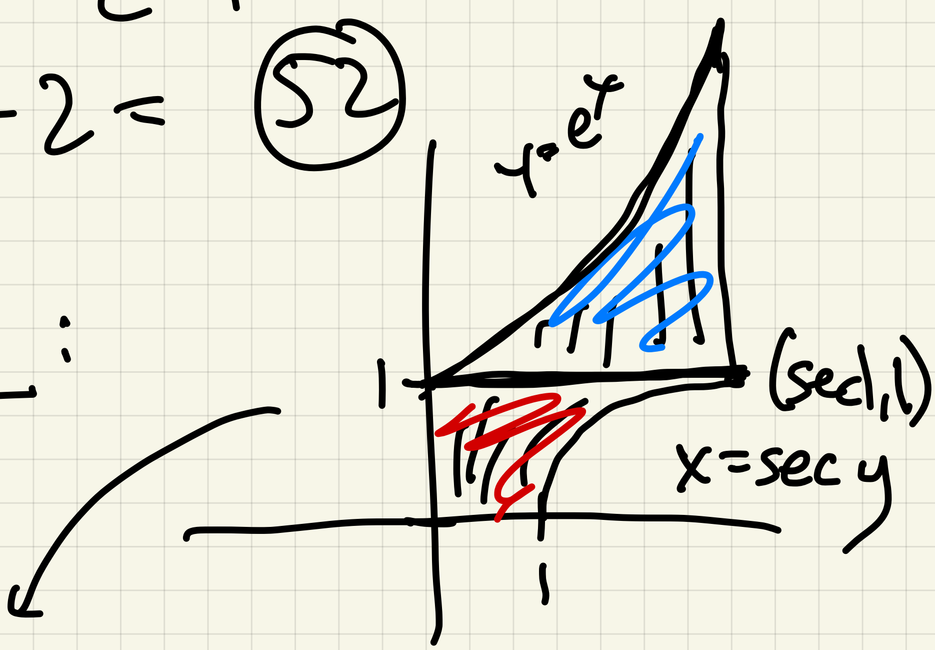
$$\int_1^9 6\cdot\frac{1}{2}\sqrt{u}\,du = \int_1^9 3\sqrt{u}\,du$$

$$\cancel{3}\cdot\frac{2}{\cancel{3}}u^{3/2}\Big|_1^9 = 2u^{3/2}\Big|_1^9 =$$

$$2\cdot 9^{3/2} - 2\cdot 1^{3/2}$$

$$2\cdot 27 - 2 = 52$$

Last example:



top $\qquad\qquad\qquad\qquad$ bottom

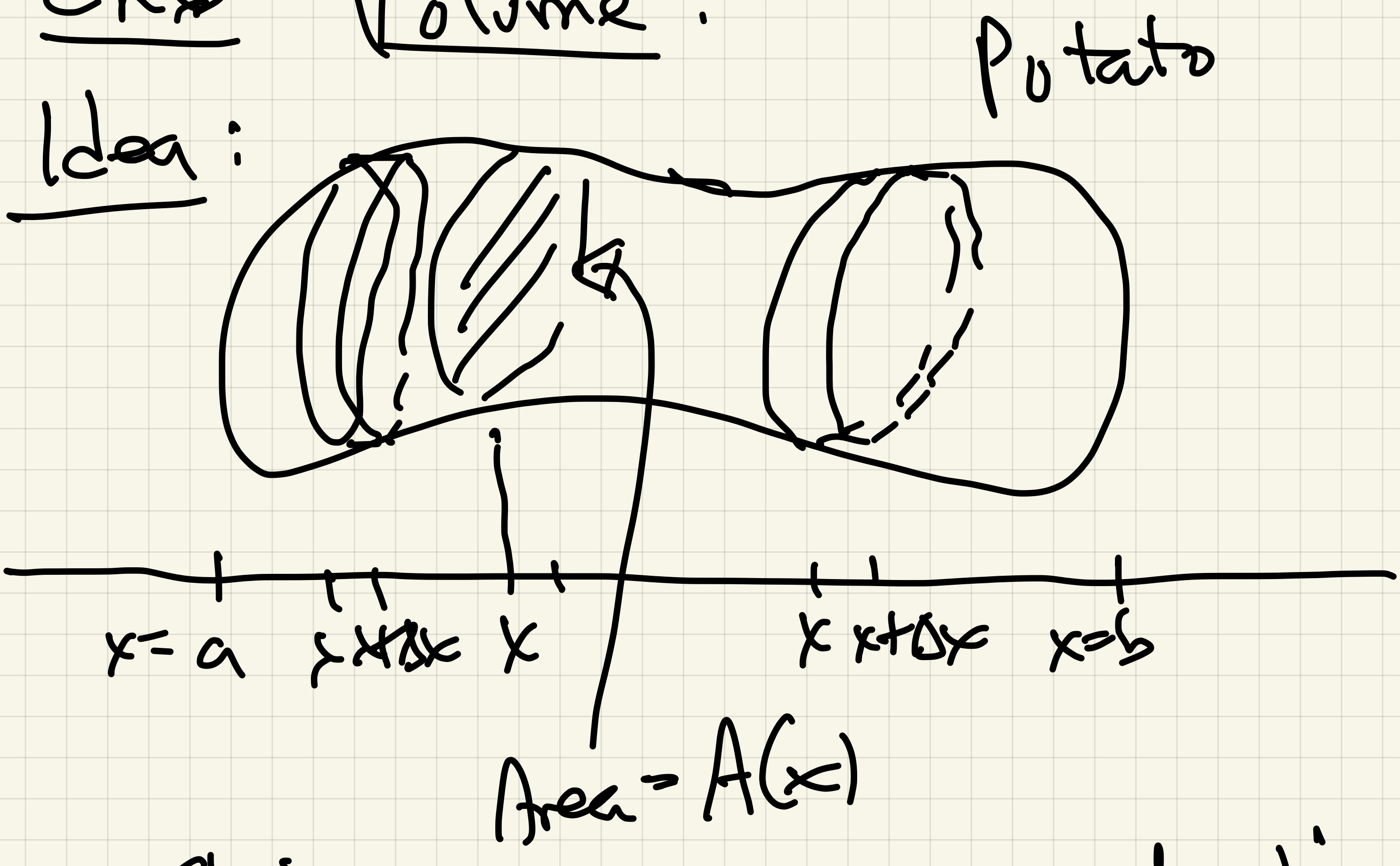
$$\int_0^{\sec 1} e^y - 1\,dx + \int_0^1 \sec y - 0\,dy$$

$$e^y - x\Big|_0^{\sec 1} + \ln|\sec y + \tan y|\Big|_0^1$$

check: $\dfrac{\sec y \tan y + \sec^2 y}{\sec y + \tan y} = \sec y$ ✓

Ch6 Volume:

Idea:



Slice potato into n equal slices

$$\text{Volume} \approx \sum_{i=1}^{n} \begin{pmatrix} \text{volume} \\ \text{of} \\ \text{each} \\ \text{slice} \end{pmatrix}$$

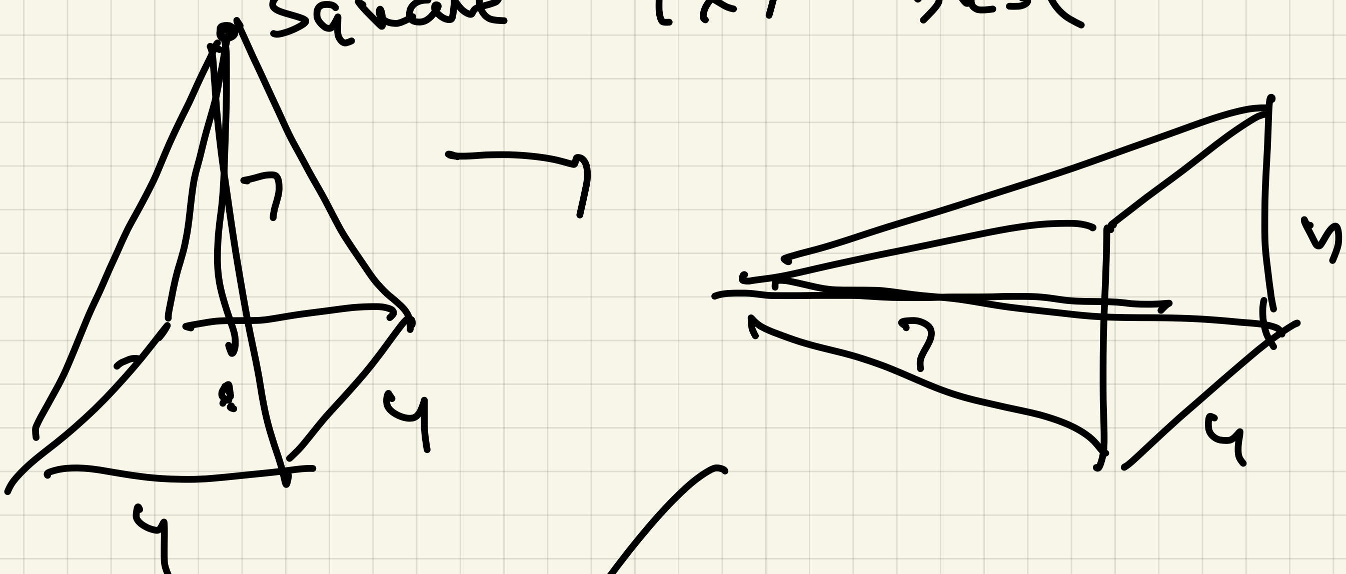
$$= \sum_{i=1}^{n} \Big(\underbrace{A(x_i)}_{\text{Area } i^{th} \text{ slice}} \cdot \underbrace{\Delta x_i}_{\text{width}} \Big)$$

$\int_0$ <u>Exact</u> volume $= V$

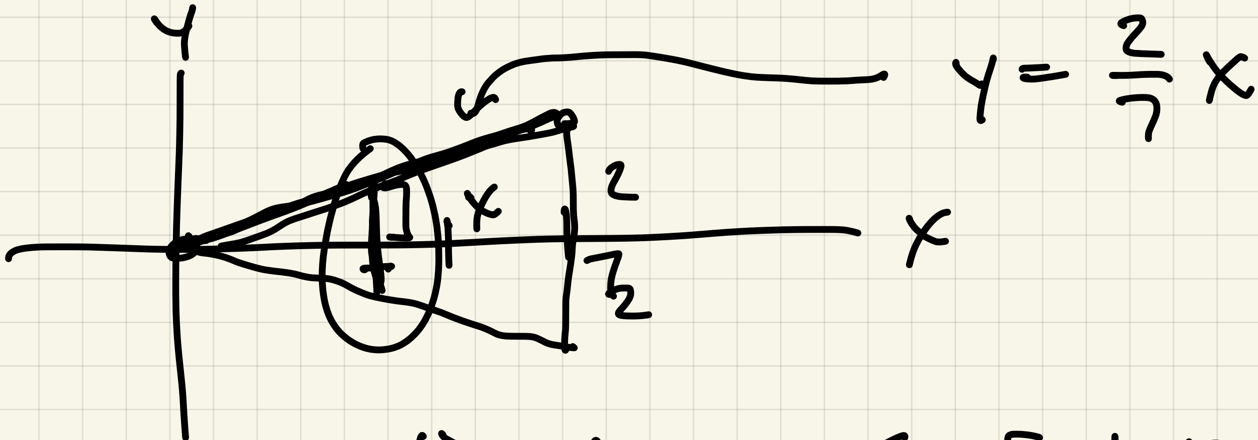
$$= \int_a^b A(x)\,dx$$

<u>Ex1</u> Pyramid of height 7

square 4×4 base



"$mx+b$"



Cross section at x is square,

side $= 2\left(\frac{2}{7}x\right) = \frac{4}{7}x$

Area of section is $\left(\frac{4}{7}x\right)^2$

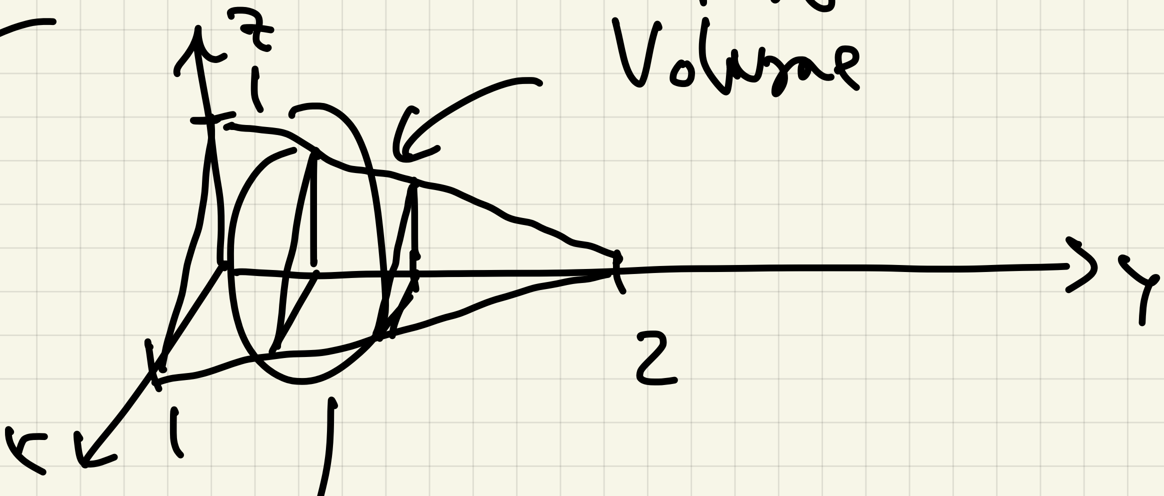
$\therefore$ Volume $= \int_0^7 \left(\frac{4}{7}x\right)^2 dx =$

$\frac{16}{49}\int_0^7 x^2 = \frac{16}{49}\left.\frac{x^3}{3}\right|_0^7 =$

$\frac{16}{49}\cdot\frac{7^3}{3} = \frac{16\cdot 7}{3} = \frac{112}{3}$

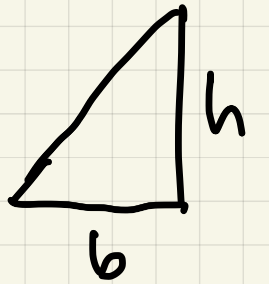
$7^2 = 49$

<u>Ex 2</u> Find Volume



Cross sectional area of △
c

$\frac{1}{2}bh$



$h =$ "$mx+b$" $=$

$-\frac{1}{2}x+1$

$b = h$

Area of △ $= \frac{1}{2}bh =$

$\frac{1}{2}\left(1-\frac{1}{2}x\right)\left(1-\frac{1}{2}x\right)$

$$V = \int_0^2 \frac{1}{2}\left(1-\frac{1}{2}x\right)^2 dx =$$

$u = 1 - \frac{1}{2}x$

$du = -\frac{1}{2}dx$

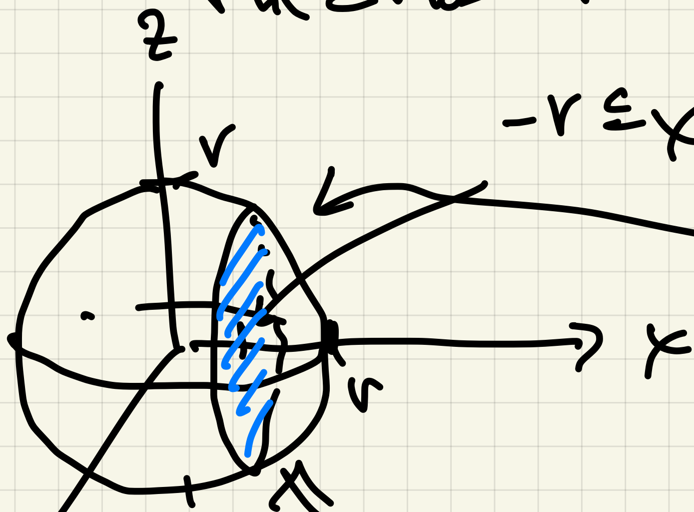
$-2du = dx$

$$\int_{u=1}^{u=0} \frac{1}{2}u^2(-2)\,du =$$

$$\int_0^1 u^2\,du = \frac{u^3}{3}\Big|_0^1 = \frac{1}{3}$$

Ex3 Volume of sphere of radius r

$-r \le x \le r$



$x^2+z^2=r^2$

Find area of cross section

circle

Radius = ?? $\longrightarrow$ $R = z = \sqrt{r^2-x^2}$

$R(x)$

Area of circle = πR^2

$$= \pi\left(\sqrt{r^2-x^2}\right)^2$$

So $$\int_{-r}^{r} \pi(r^2-x^2)\,dx =$$

$$\pi\left(r^2x - \frac{x^3}{3}\right)\Big|_{-r}^{r} =$$

$$\pi\left(\left(r^3 - \frac{r^3}{3}\right) - \left(-r^3 - \frac{(-r)^3}{3}\right)\right) =$$

$$\pi\left(r^3 - \frac{r^3}{3} + r^3 - \frac{r^3}{3}\right)$$

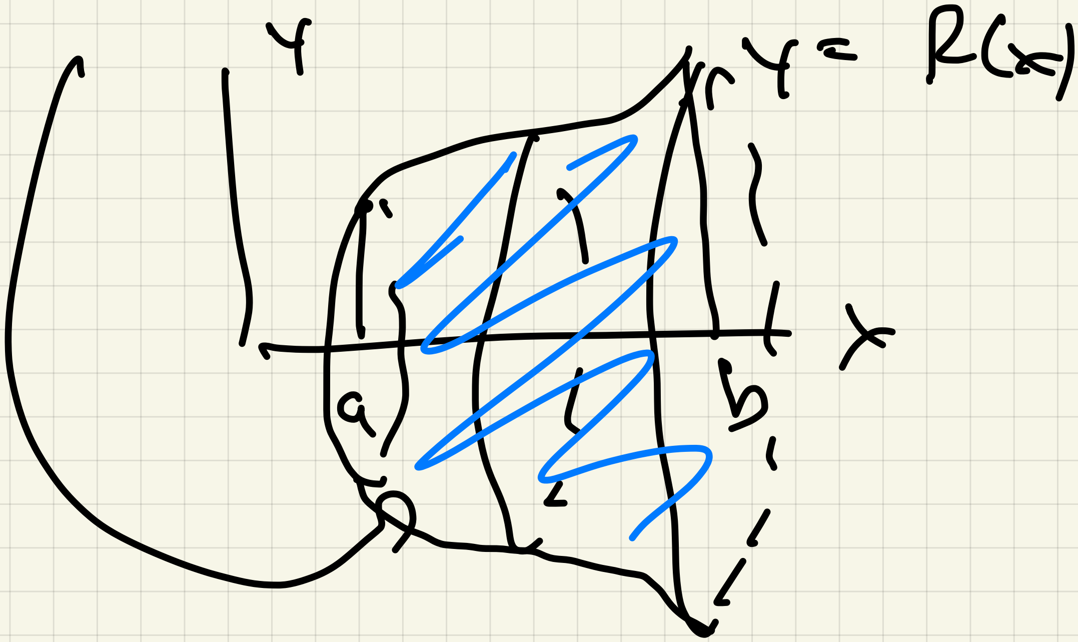
$$\pi\left(2r^3 - \frac{2r^3}{3}\right) = \pi\frac{4}{3}r^3 =$$

$$\boxed{V = \frac{4}{3}\pi r^3}$$

Ex3 illustrates more general

Disk method If cross sections are circles of radius $R(x)$, then the volume of solid of revolution is

$$V = \int_a^b \pi\,(R(x))^2\,dx$$

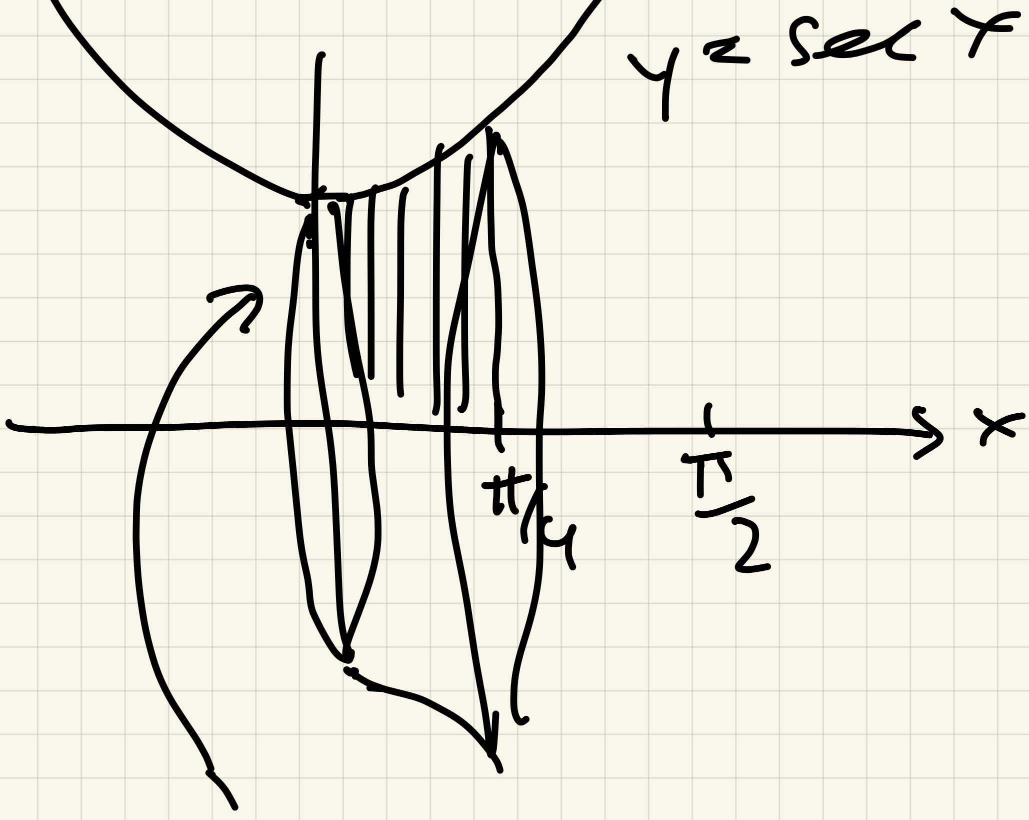


Ex 4 Find volume of revolution

of the region $0 \le x \le \pi/4$,

$\boxed{0 \le y \le \sec x}$

about x-axis



$$V = \int_0^{\pi/4} \pi (\sec x)^2 \, dx =$$

$$\int_0^{\pi/4} \pi \sec^2 x \, dx =$$

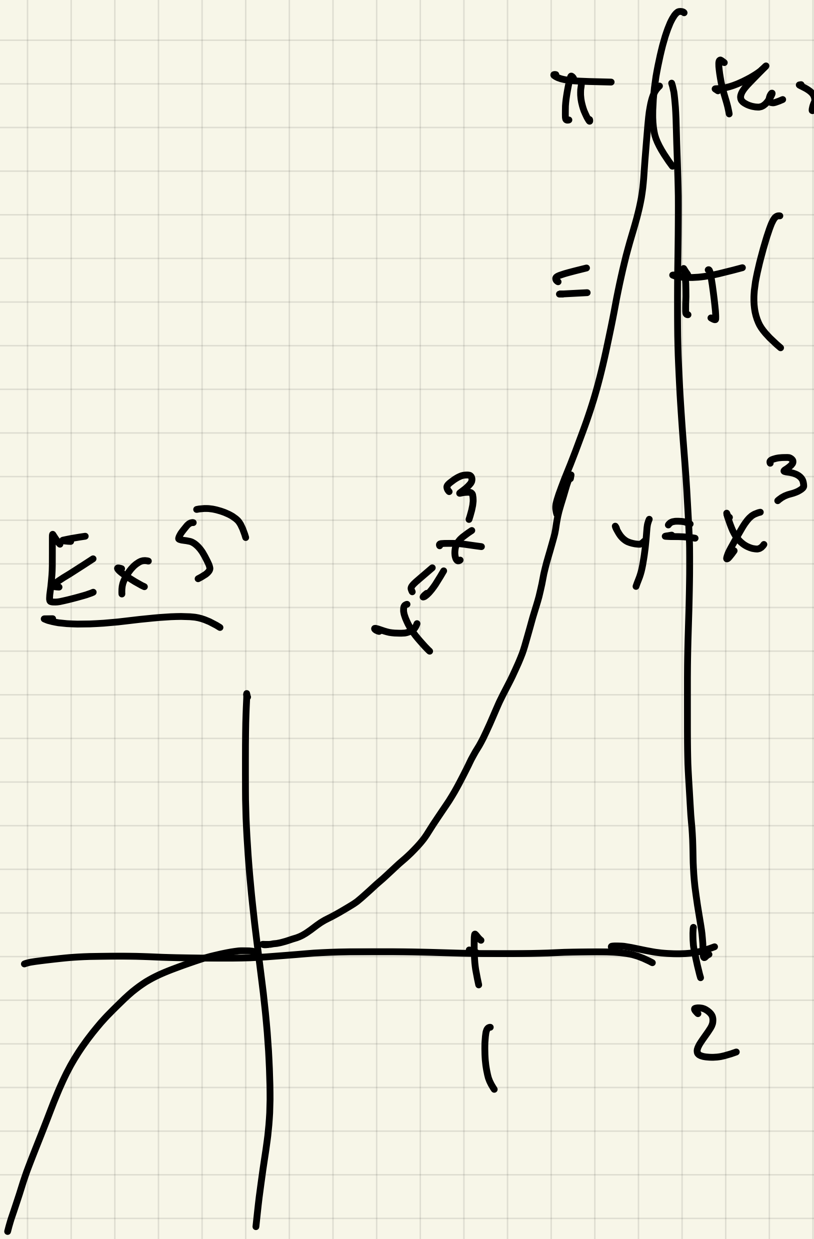
$$\pi \tan x \Big|_0^{\pi/4} =$$

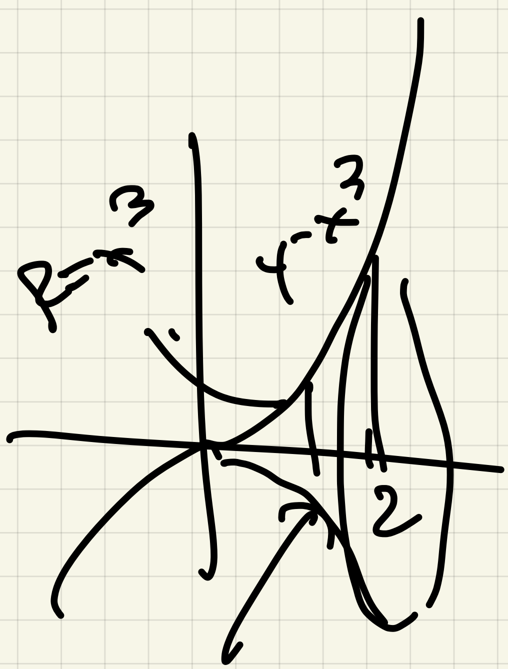
$$\pi \left(\tan \tfrac{\pi}{4} - \tan 0 \right)$$

$$= \pi (1 - 0)$$

$$= \pi$$

<u>Ex 5</u>





find volume

$$V = \int_0^2 \pi (\text{radius})^2 dx =$$

$$\int_0^2 \pi (x^3)^2 dx =$$

$$\int_0^2 \pi x^6 dx = \pi \frac{x^7}{7} \Bigg|_0^2 =$$

$$\pi \left(\frac{128}{7} \right) = \frac{128\pi}{7}$$