HW 13

Section 8.7 # $\underbrace{2, 9, 14, 17, 20, 33}_{10}, \underbrace{72, 77}_{15}, \underbrace{A}_{10}$

$\boxed{\text{Total} = 100}$

$\boxed{2}$ $\int_1^\infty \frac{dx}{x^{1.001}}$ $\quad p = 1.001 > 1 \Longrightarrow \frac{1}{p-1} = \frac{1}{.001} = 1000$

$\boxed{9}$ $\int_{-\infty}^{-2} \frac{2\,dx}{x^2-1} = \int_{-\infty}^{-2} \frac{1}{x-1} - \frac{1}{x+1}\,dx =$

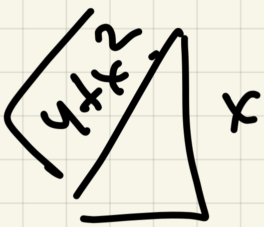
partial fractions

$\lim_{a \to -\infty} \ln\left|\frac{x-1}{x+1}\right|\Big|_a^{-2} = \lim_{a \to -\infty} \ln 3 - \ln\left|\frac{a-1}{a+1}\right|$

$= \ln 3 - \ln 1 = \ln 3.$

$\boxed{14}$ $\int_{-\infty}^{\infty} \frac{x\,dx}{(x^2+4)^{3/2}}$ $\quad x = 2\tan\theta$, $\quad dx = 2\sec^2\theta\,d\theta$

$\int \frac{2\tan\theta \, 2\sec^2\theta}{(2\sec\theta)^3} = \int \frac{\tan\theta}{2\sec\theta} = \int \frac{1}{2}\sin\theta =$



$-\frac{1}{2}\cos\theta = -\frac{1}{2}\left(\frac{2}{\sqrt{4+x^2}}\right)$, so

$$\int_0^\infty \frac{x}{(x^2+4)^{3/2}} = \lim_{b\to\infty} \int_0^b \frac{x}{(x^2+4)^{3/2}} =$$

$$\lim_{b\to\infty} \left.\frac{-1}{\sqrt{4+x^2}}\right|_0^b = \lim_{b\to\infty} \frac{-1}{\sqrt{4+b^2}} + \frac{1}{\sqrt{4}} = \frac{1}{2}$$

Similarly $\int_{-\infty}^0 \frac{x}{(x^2+4)^{3/2}} = -1/2$, so $\int_{-\infty}^\infty = 0$

$\boxed{17}$ $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$ $\quad x = u^2$, $dx = 2u\,du$, $\sqrt{x} = u$

$$\int \frac{2u\,du}{(1+u^2)u} = \int \frac{2}{1+u^2} = 2\arctan u = 2\arctan\sqrt{x} + C$$

so

$$\lim_{b\to\infty} \int_0^b \frac{dx}{(1+x)\sqrt{x}} = \lim_{b\to\infty} 2\tan^{-1}\sqrt{b} - 0 =$$

$$2 \cdot \pi/2 = \pi$$

$\boxed{20}$ $\int_0^\infty \frac{16\tan^{-1}x}{1+x^2}dx$ $\quad u = \tan^{-1}x$, $du = \frac{1}{1+x^2}$

$$= \int 16u\,du = 8u^2 = 8(\tan^{-1}x)^2, \text{ so}$$

$$\int_0^\infty \frac{16\tan^{-1}x}{1+x^2} = \lim_{b\to\infty} \left. 8(\tan^{-1}x)^2 \right|_0^b =$$

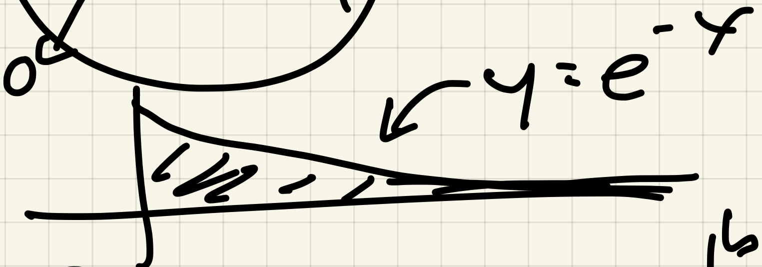
$$\lim_{b\to\infty} 8(\tan^{-1}b)^2 = 8\left(\frac{\pi}{2}\right)^2 = 2\pi^2$$

$\boxed{33}$ $\int_{-1}^{\infty} \frac{d\theta}{\theta^2+5\theta+6} = \int_{-1}^{\infty} \frac{d\theta}{(\theta+2)(\theta+3)} =$

$\int_{-1}^{\infty} \frac{1}{\theta+2} - \frac{1}{\theta+3} = \lim_{b\to\infty} \ln\left|\frac{\theta+2}{\theta+3}\right|\Big|_{-1}^{b} =$

$\lim_{b\to\infty} \underbrace{\ln\left|\underbrace{\frac{b+2}{b+3}}_{\to 1}\right|}_{0} - \ln\left|\frac{1}{2}\right| = -\ln\left(\frac{1}{2}\right) = \ln 2.$

$\boxed{72}$



$M = \int_0^{\infty} e^{-x} = \lim_{b\to\infty} -e^{-x}\Big|_0^b = \lim_{b\to\infty} -e^{-b} + 1 \to 1$

$M_y = \int_0^{\infty} xe^{-x} = \lim_{b\to\infty} -xe^{-x} - e^{-x}\Big|_0^b =$

$\lim_{b\to\infty} (-b-1)e^{-b} - (-1) = \lim_{b\to\infty} 1 - \frac{(b+1)}{e^b} =$

$\lim_{b\to\infty} 1 - \frac{1}{e^b} = 1$

$M_x = \int_0^{\infty} \frac{(e^{-x})^2 - 0}{2} = \lim_{b\to\infty} \int_0^b \frac{e^{-2x}}{2}\,dx =$

$\lim_{b\to\infty} \frac{-e^{-2x}}{4}\Big|_0^b = \lim_{b\to\infty} -\frac{e^{-2b}}{4} + \frac{1}{4} = \frac{1}{4}$

So $(\bar{x},\bar{y}) = \left(\frac{1}{1}, \frac{1/4}{1}\right) = (1, 1/4)$

$\boxed{77}$

(a) Area $= \int_1^\infty \frac{1}{x^2}\,dx = \frac{1}{2-1} = 1$ by Ex 3

(b) x-axis : $\int_1^\infty \pi\left(\frac{1}{x^2}\right)^2 dx = \pi\int_1^\infty \frac{1}{x^4}\,dx =$ Ex 3

$\pi\left(\frac{1}{4-1}\right) = \frac{\pi/3}{}$

y-axis $\int_1^\infty 2\pi x\left(\frac{1}{x^2}\right)dx =$

$2\pi\int_1^\infty \frac{1}{x}\,dx$ diverges (by Ex 3).

$\boxed{A.}$ $\int_3^\infty \frac{dx}{(x^2-1)(x^2-4)} = \int_3^\infty \frac{dx}{(x-1)(x+1)(x-2)(x+2)}$

$\frac{1}{(x-1)(x+1)(x-2)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-2} + \frac{D}{x+2}$

$1 = A(x+1)(x^2-4) + B(x-1)(x^2-4) + C(x+2)(x^2-1) + D(x-2)(x^2-1)$

$x=1 \Rightarrow A = -1/6$, $x=-1 \Rightarrow B = 1/6$

$x=2 \Rightarrow C = 1/12$, $x=-2 \Rightarrow D = -1/12$

$\int_3^\infty = \lim_{b\to\infty}\int_3^b \frac{-1/6}{x-1} + \frac{1/6}{x+1} + \frac{1/12}{x-2} - \frac{1/12}{x+2} =$

$$\lim_{b\to\infty} \frac{1}{6}\ln\left|\frac{x+1}{x-1}\right| + \frac{1}{12}\ln\left|\frac{x-2}{x+2}\right| \Big|_3^b =$$

$$\lim_{b\to\infty} \frac{1}{6}\ln\left|\frac{b+1}{b-1}\right| + \frac{1}{12}\ln\left|\frac{b-2}{b+2}\right| - \frac{\ln 2}{6} - \frac{\ln \frac{1}{5}}{12}$$

$$= \frac{\ln 5}{12} - \frac{\ln 2}{6} = \frac{\ln 5}{12} - \frac{\ln 4}{12} = \frac{1}{12}\ln \frac{5}{4}$$