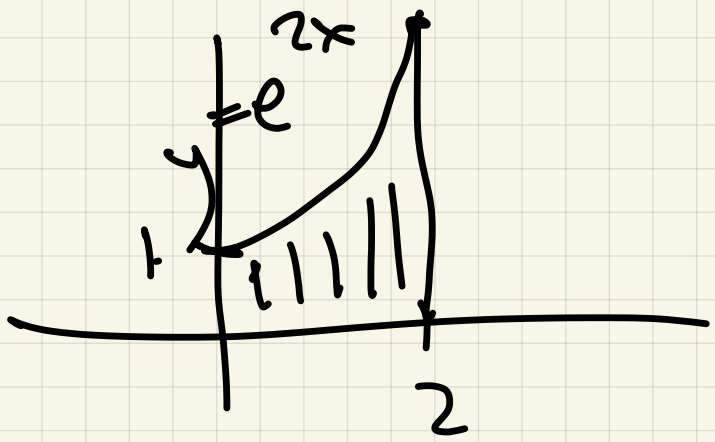
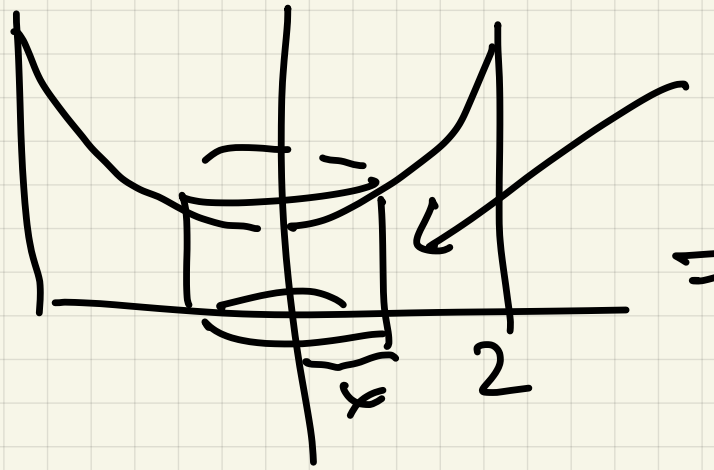
2/9/Calc2 Quiz 4



1. revolve y-axis $\ln y = 2x$

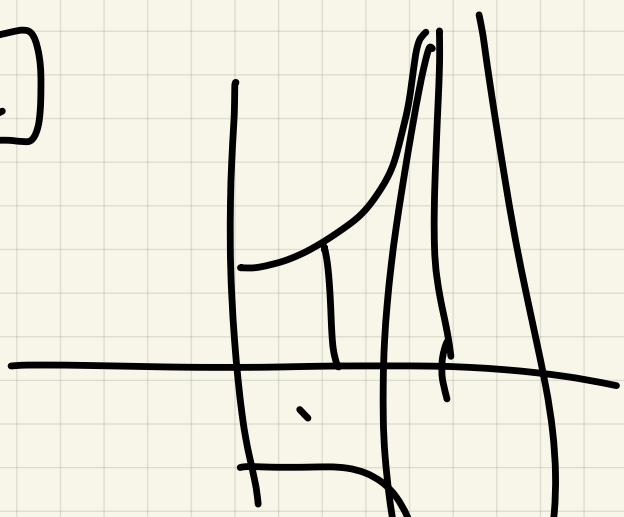


$$\Rightarrow V = \int_0^2 2\pi x e^{2x} dx$$

Washers:

$$V = \int_0^1 \pi(2^2 - 0^2)\, dy + \int_1^{e^4} \pi\left(2^2 - \left(\frac{\ln y}{2}\right)^2\right) dy$$

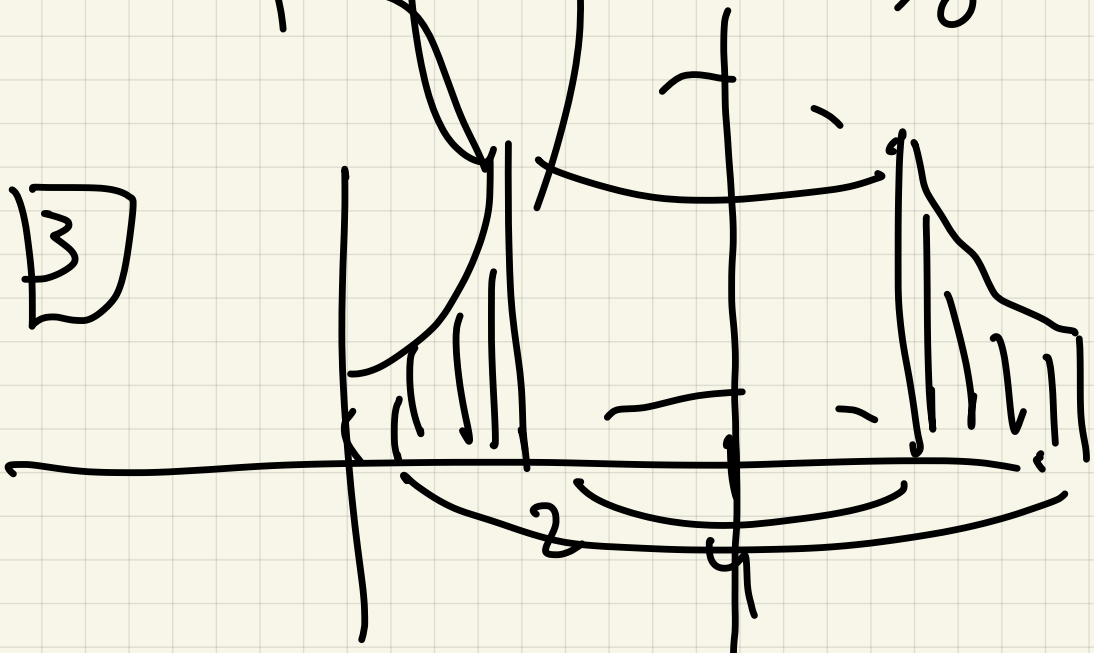
2



disks:

$$V = \int_0^2 \pi \left((e^{2x})^2\right) dx$$

3

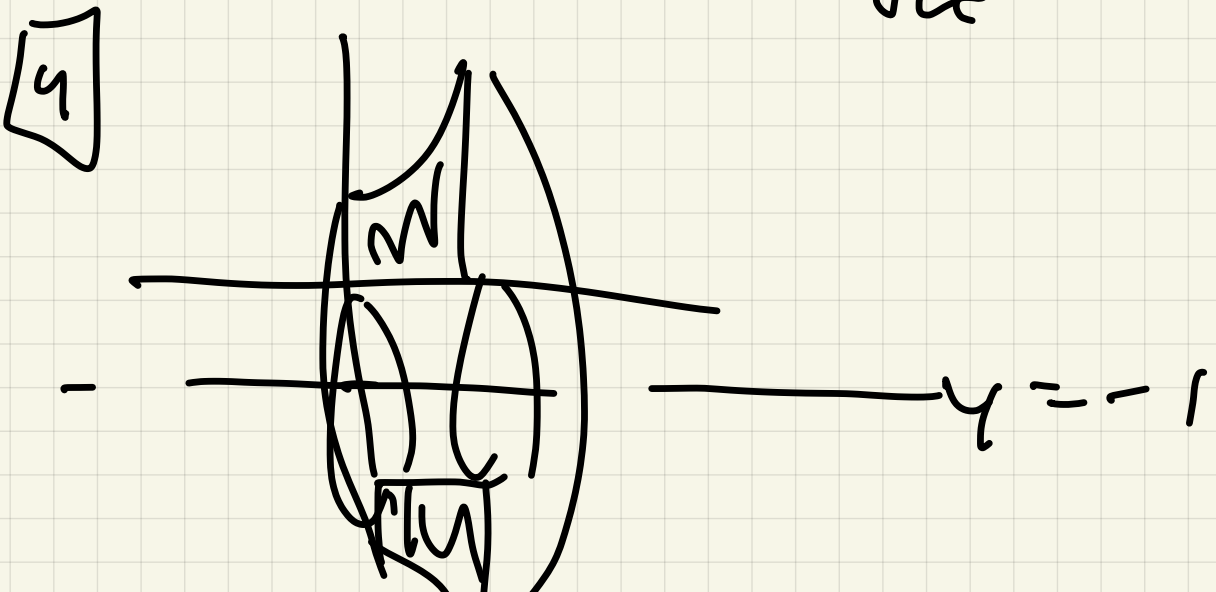


Shells:

$$\int_0^2 2\pi (4 - x) e^{2x} dx$$

rad

4



$$\int_0^2 \pi\left(\left(e^{2x}+1\right)^2 - 1^2\right) dx$$

Last time: Work: $W = F \cdot d$

F variable: $\int_a^b F(x)\,dx$

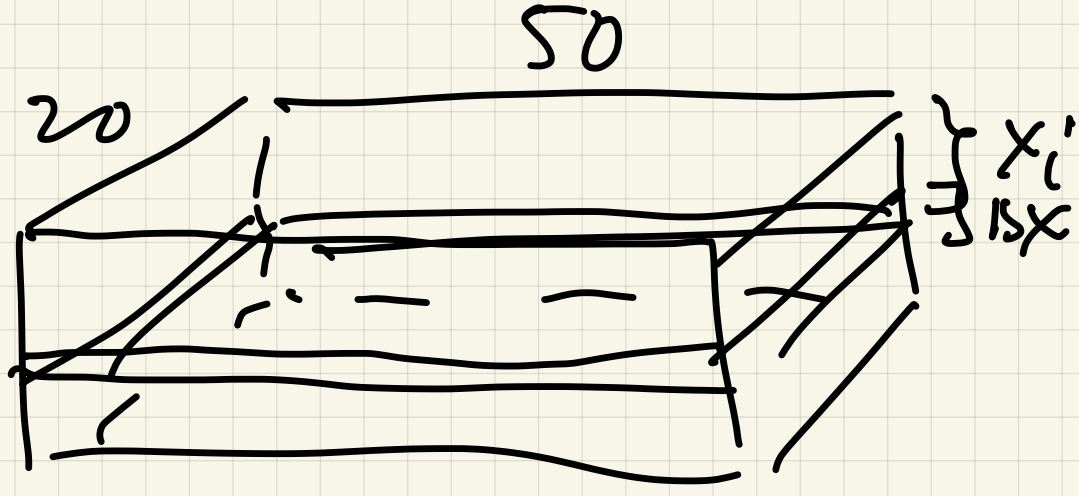
Spring problem

Chain problem

Tank problem

Ex 0

Pool of water



Water weighs 62.4 $(lbs)/ft^3$

weight of slab water at depth x_i is

$$F \quad \underbrace{50 \times 20 \times \Delta x}_{\text{volume}} \cdot \underbrace{62.4}_{\text{density}}$$

dist x_i (measured from top)

$$\therefore W \approx \sum_{i=1}^{n} 50 \times 20 \times 62.4 \times x_i \Delta x$$

exact work:

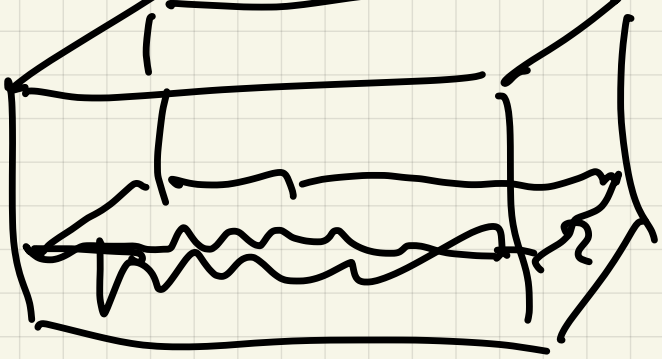
$$W = \lim_{n \to \infty} (\quad) = \int_0^5 \underline{50 \times 20 \times 62.4 \times x} \, dx$$

$$= \int_0^5 62400 \, x \, dx =$$

$$62400 \frac{x^2}{2} \Big|_0^5 =$$

(4) Variations:

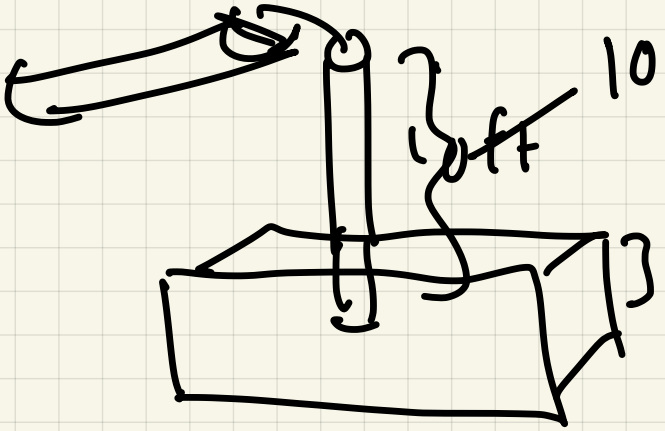
(a) Pool is filled to 2 feet from line?



$$\int_3^5 62400\, x\, dx$$

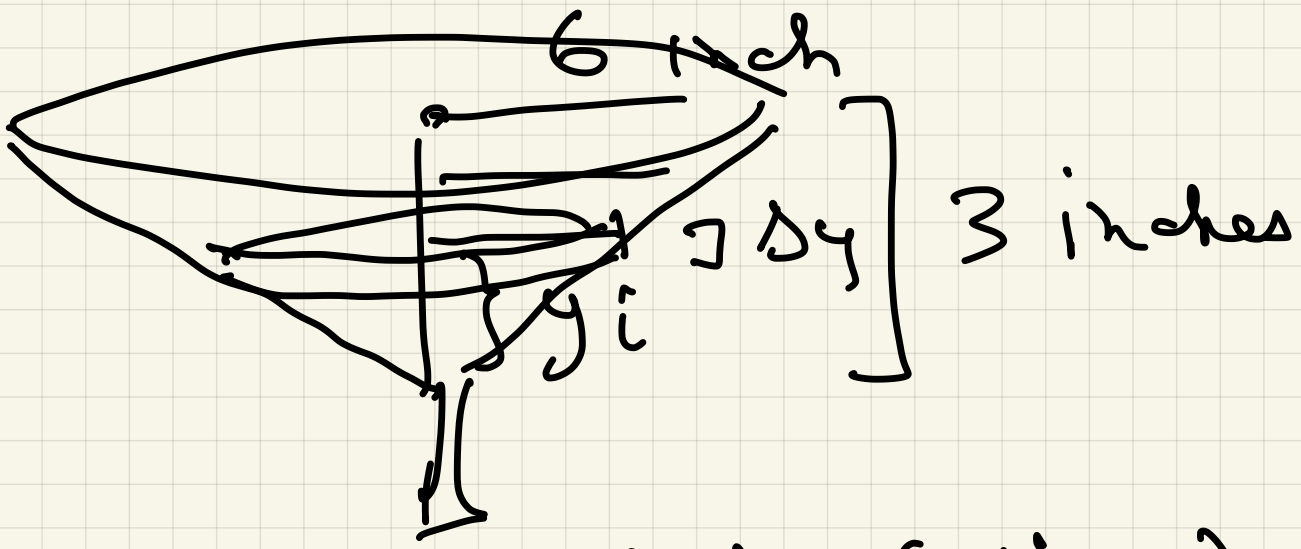
$x = 3 \sim 2$ ft

(b) We also pump water to 10 ft over top of pool



$$\int_3^5 62400\,(10+x)\,dx$$

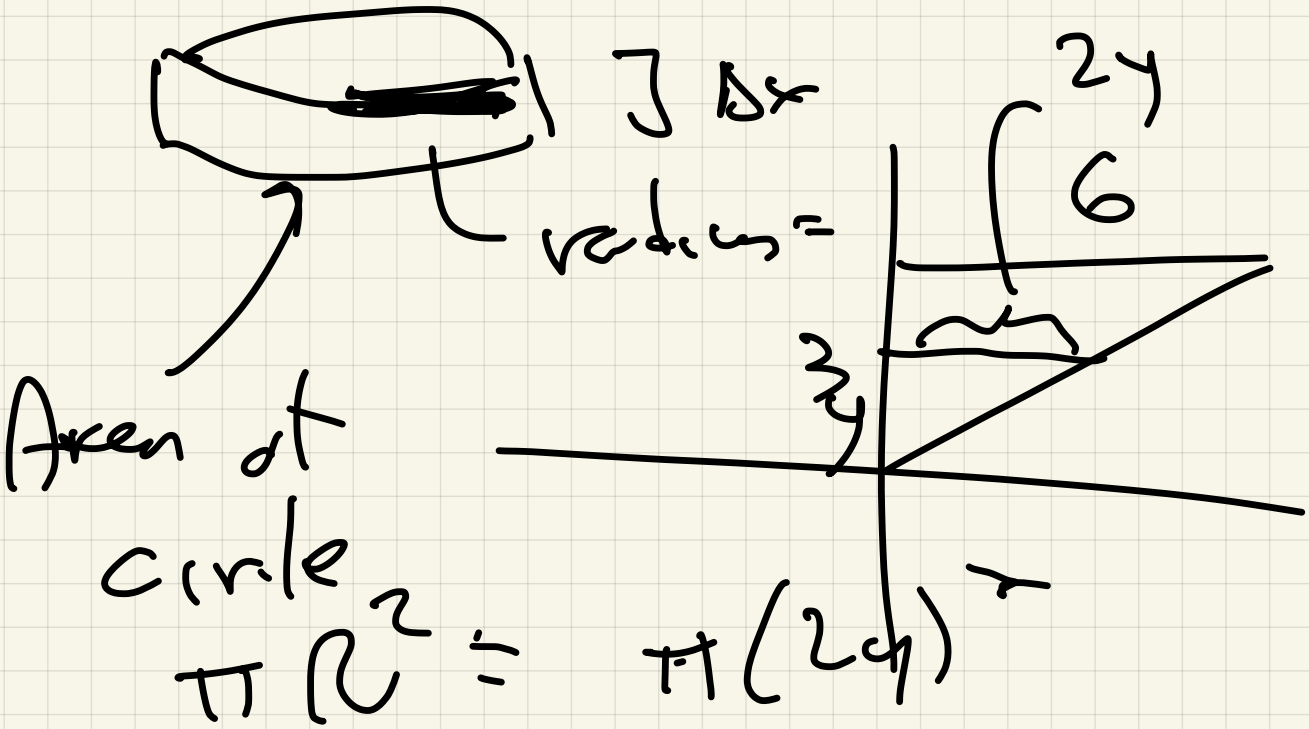
Ex2 A cone shaped glass with drink of density .02 lb/in^3. Find work done to empty glass



y_i = ht of slab of liquid
(measure from bottom)

slab of liquid

(Volume of slab) × (density)
(.02)



Volume $\pi(2y_i)^2 \Delta y$

$F = Wt$ $\pi(2y_i)^2 \Delta y\,(.02)$

dist $3 - y_i$

$W \approx \sum \pi(2y_i)^2 (3-y_i)(.02)\Delta y$

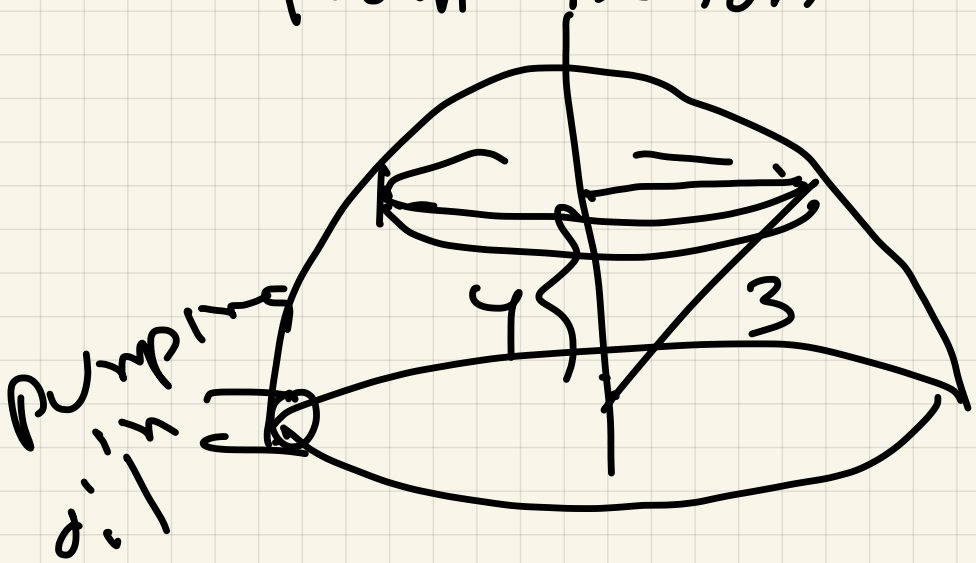
$$W = \int_0^3 \pi(2y)^2 (3-y)(.02)\,dy$$

$$= \frac{27\pi}{50} \text{ in-lbs}$$

(b) What is work to move liquid to lt 10 inches through straw?

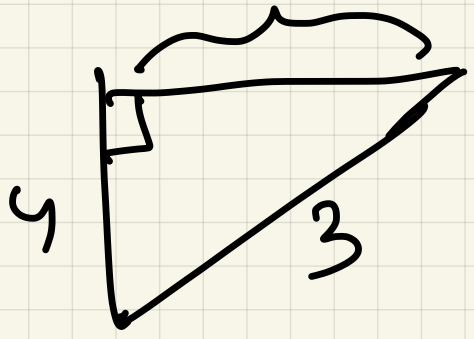
$$\int_0^3 \pi(2y)^2 (13-y)(.02)\,dy$$

Ex2 What is work done to pump a half dome of radius 3m full of oil with density 200 kg/m³ from bottom?



Measure y from bottom:

radius = r



$y^2 + r^2 = 3^2$

$r = \sqrt{9 - y^2}$

area of slab of oil

$\pi(r^2) = \pi\left(\sqrt{9-y^2}\right)^2$

$= \pi(9-y^2)$

Volume $\pi(9-y^2)\Delta y$

Wt $\pi(9-y^2)\Delta y\,(20)$

dist y

$\therefore$ ~~F~~

$$W = \int_0^3 \pi(9-y^2)(20)\,y\,dy$$

$$= 127.189.81 \text{ N-m} = \text{J}$$

General:

1. draw picture
2. Set variable y from vertical position
3. Find end points that

corresponds to liquid

(4) Find cross section area $A(y)$

(5) Find distance to move slab $D(y)$ of liquid

$$W = \int_a^b \rho \cdot A(y) \cdot D(y)\, dy$$

density

density