2/5 | Calc2 Work

Idea: $W = F \cdot d$

force, distance

Units: ft-lbs / in-lbs / N-m = J

Ex1 Find Work done to lift 200 lb to height of 3 ft

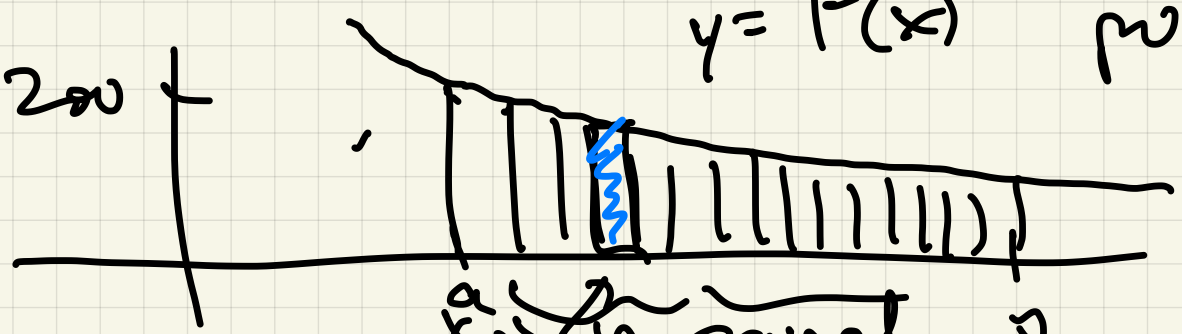
(a) $W = 200 \times 3 = 600$ ft-lbs

(b) Work to lift 200 lb up to 4-miles ??

Subtle: force decreases as distance grows

Variable Force:

$y = F(x) =$ force at position



x_i in equal

pieces

Work done from x_i to x_{i+1}

is roughly $\Delta W_i = F(x_i) \cdot \Delta x_i$

force $\quad x_{i+1} - x_i$

Total work is

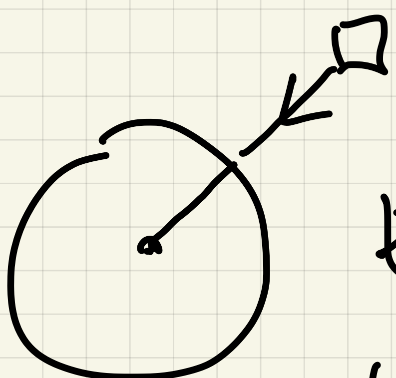
$$W \approx \sum_{i=1}^{n} \Delta W_i \approx \sum_{i=1}^{n} F(x_i)\, \Delta x_i$$

Exact work: $W = \lim_{n \to \infty} \sum_{i=1}^{n} F(x_i) \Delta x_i$

$$= \int_a^b F(x)\, dx$$

Physics: $F(x)$ proportional to $\frac{1}{x^2}$,

x = dist to center of earth



i.e. $F(x) = \frac{C}{x^2}$, (const)

Radius of earth is 4000 miles

$$200 \text{ lbs} = \frac{C}{(4000)^2} \Rightarrow$$

$$C = 200 \cdot (4000)^2 = 3200000000$$
$$= 3.2 \times 10^9$$

(b) $$W = \int_{4000}^{4004} \frac{(3.2 \times 10^9)}{x^2}\, dx =$$

7.992008 mile-lbs

$$\left(W = F \cdot d = 200 \cdot 4 = 800 \text{ doc} \right)$$

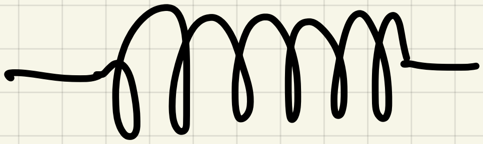
(c) Work to move weight 4000 miles:

$$\int_{4000}^{8000} \frac{3.2 \times 10^9}{x^2}\, dx = 400{,}000 \text{ mile-lbs}$$

$$\left(\underline{\text{Note}}: W = F \cdot d = 200 \cdot 4000 = 800{,}000 \right)$$

<u>Spring Problems</u>

<u>Hooke's law</u>:



rest length ; stretch it ;

If a spring is pulled/pushed x units beyond natural.

The spring recoils with a force = $F = kx$

Ex1: A force of 60 lbs holds a spring at 2 ft beyond rest length.

(a) Find ~~spring~~ spring constant k

(b) Work done to stretch spring from rest length to 3 ft further.

(c) compress spring to 1 ft shorter

(d) Walk to stretch spring from 1-ft compressed out to 3 ft beyond rest length

(a) $F = kx \rightarrow k = 30 \text{ lbs/ft}$

$F = 60 \text{ lbs}$, $x = 2 \text{ ft}$

(b) $W = \int kx\,dx$

$$= \int_0^3 30x\,dx =$$

$$15x^2\Big|_0^3 = 15 \cdot 9 = 135 \text{ ft-lbs}$$

(c) $W = \int_0^{-1} 30x\,dx = 15x^2\Big|_0^{-1} =$

15 ft-lbs

(d) $W = \int_{-1}^{3} 30x\,dx = 15x^2\Big|_{-1}^{3}$

$135 - 15 = 120$

Ex2 150 ft-lbs work is required to stretch a spring to 6 in beyond its natural length.

(a) Find spring constant

(b) Work to stretch spring to 12 inches

(c) 50 ft-lbs work compresses spring by how much?

(a) $W = \int kx\,dx$

$$\cancel{150} = \int_0^6 kx\,dx = \tfrac{1}{2}kx^2 \Big|_0^6 =$$

$$\| \qquad 18k$$

ft-lbs

$\|$

in lbs $\left(\frac{ft}{in}\right)$

150 ft-lbs =

12×150

1800 in-lbs

$$1800 = \int_0^6 kx\,dx = \frac{1}{2}kx^2\Big|_0^6 = 18k$$

$$\boxed{k = 100 \ \text{lbs/inch}}$$

(b) $\displaystyle\int_0^{12} 100x\,dx$

$$= 50x^2\Big|_0^{12} = 50 \cdot 144 = 7200 \ \text{in-lbs}$$

(c)

50 ft-lbs

$\parallel$

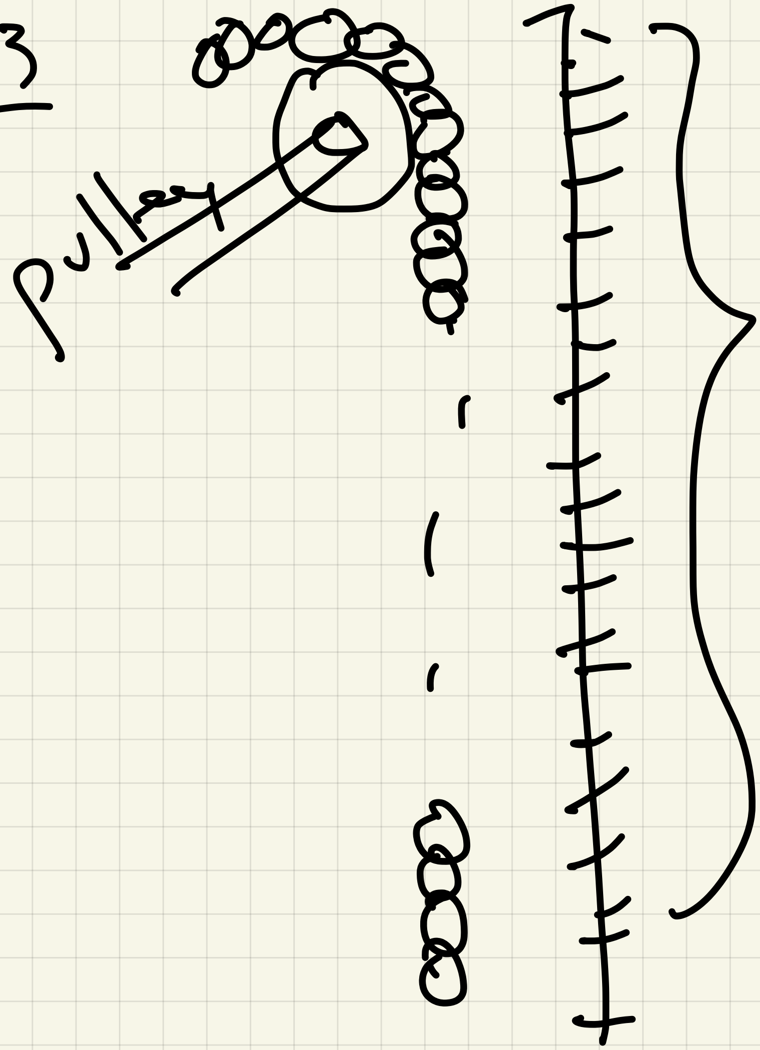
$$600 \ \text{in-lbs} = \int_0^{-\ell} 100x\,dx \Rightarrow$$

$$\underbrace{600 = 50x^2\Big|_0^{-\ell} = 50\ell^2}$$

$$\ell^2 = 12 \Rightarrow \ell = \sqrt{12} \ \text{inches}$$

<u>Varying distance</u>

Ex 3



100 ft. of chain, weighs 5 lbs/ft.

(a) How much work to crank chain over top?

(b) to halfway mark to 50 ft?

Idea: ~~Consider a~~

Break chain into n equal pieces of length $\Delta x = \frac{100}{n}$

How much work to move

one segments to top?

weighs

$5 \cdot \Delta x$ lbs

$5\,lb/ft \cdot \Delta x\,ft =$

$5\Delta x$ lbs

dist $= x_i$

$$\Delta W_i = \underbrace{x_i}_{\text{dist}} \cdot \underbrace{5\Delta x}_{\text{force}}$$

<u>Total</u> <u>work</u>: $W \approx \sum_{i=1}^{n} \Delta W_i$

$$\approx \sum_{i=1}^{n} x_i 5\Delta x$$

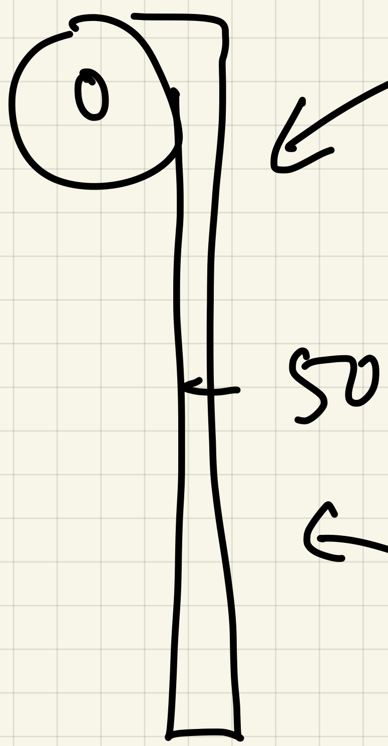
<u>Exact work</u>:

$$\lim_{n \to \infty} \sum_{i=1}^{n} 5x_i \Delta x = \int_0^{100} 5x\,dx$$

(a) $$\int_0^{100} 5x\,dx = \frac{5x^2}{2}\Big|_0^{100} =$$

$$\frac{50000}{2} = 25000 \text{ ft-lbs}$$

(b)



$$W_{top} = \int_0^{50} 5x\,dx$$

$$= 6250 \quad +$$

$$W_{bottom} = \int_0^{50} 5 \cdot 50\,dx$$

$$= 12500$$

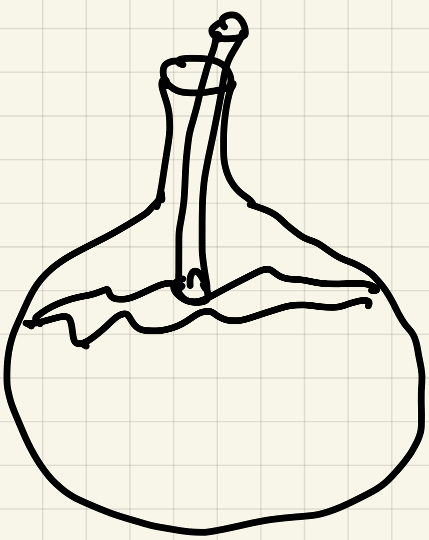
total work: $12500 + 6250 =$

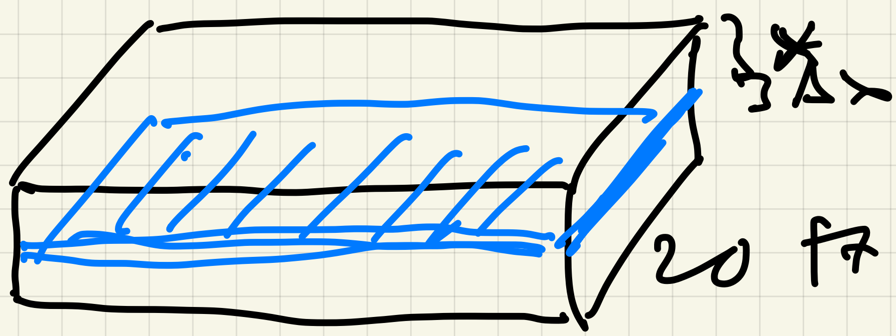
18,750 ft-lbs

Varying distance & weight

Ex1 water weighs 62.4 lbs/ft^3

Find work to pump water out of a swimming pool





A slab/layer of water at depth x and thickness Δx

Slab of water weighs = Volume x density

$$= \underbrace{(50 \times 20 \times \Delta x)}_{\text{volume}} \underbrace{(62.4)}_{\text{wt}} \underbrace{x}_{\text{dist}}$$

distance to move is x

$$\text{so} \quad W = \int_0^5 \underline{(50 \times 20)}\,\underline{(62.4)}\, x \, dx$$

$$62400 \int_0^5 x \, dx$$

$$62400 \left. \frac{x^2}{2} \right|_0^5 = \frac{25}{2} \cdot 62400$$

$$= 780{,}000 \text{ ft} \cdot \text{lbs}$$