2/26/Calc 2 Quiz 8 avg 80

$$\int_0^1 \underbrace{x^2}_{u} \underbrace{e^{4x}\,dx}_{dv} =$$

$$\boxed{du = 2x\,dx \qquad v = \frac{1}{4}e^{4x}}$$

$$\frac{1}{4}x^2e^{4x} - \int \frac{1}{4}\cdot 2x\,e^{4x}\,dx$$

$$\frac{1}{4}x^2e^{4x} - \frac{1}{2}\int \underbrace{x}_{u}\,\underbrace{e^{4x}\,dx}_{dv}$$

$$\boxed{du = dx \qquad v = \frac{1}{4}e^{4x}}$$

$$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,dx$$

$$\left(\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\right)$$

$$\frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x}\Big|_0^1 =$$

$$\left(\frac{1}{4}e^4 - \frac{1}{8}e^4 + \frac{1}{32}e^4\right) - \frac{1}{32}$$

$$\frac{5e^4 - 1}{32}$$

<u>Last time</u> $\qquad x = \sin\theta$

① $\displaystyle\int \sqrt{1-x^2}\,dx = \frac{1}{2}\arcsin x + \frac{1}{2}x\sqrt{1-x^2} + C$

<u>Aside</u> <u>Another way</u>:

$$\int \underbrace{\sqrt{1-x^2}}_{u}\,\underbrace{dx}_{dv}$$

$$du = \frac{-x}{\sqrt{1-x^2}} \qquad v = x$$

$$(1-x^2)^{1/2} \to \frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)$$

$$x\sqrt{1-x^2} + \underbrace{\int \frac{+x^2}{\sqrt{1-x^2}}}$$

$$\int \frac{x^2 - 1 \;\; {}''{+1}}{\sqrt{1-x^2}} =$$

$$\int -\frac{\left(\sqrt{1-x^2}\right)^2}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}$$

$$\underline{\underline{So}} = -\int \sqrt{1-x^2} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\int \sqrt{1-x^2}\,dx = x\sqrt{1-x^2} - \int \sqrt{1-x^2} + \arcsin x$$

Fold back

$$2\int \sqrt{1-x^2} = x\sqrt{1-x^2} + \arcsin x$$

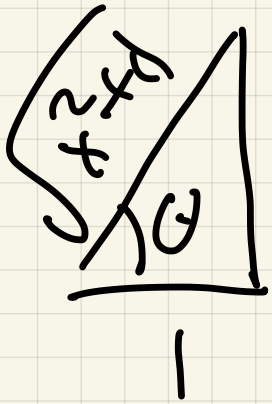
$$So \int \sqrt{1-x^2} = \frac{x\sqrt{1-x^2} + \arcsin x}{2} + C$$

$$\boxed{x = \tan\theta}$$

② $$\int \sqrt{1+x^2}\,dx = \int \sec^3\theta\,d\theta =$$

Fold back

$$\frac{\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|}{2} + C$$



$\tan\theta = x$

$\sec\theta = \sqrt{x^2+1}$

$$\frac{x\sqrt{x^2+1} + \ln\left|\sqrt{x^2+1}+x\right|}{2} + C$$

<u>Suggests a method:</u>

trig substitution to remove $\sqrt{\ }$ sign

(A) $\sqrt{a^2-x^2} \Rightarrow x = a\sin\theta \quad dx = a\cos\theta\, d\theta$

$$\sqrt{a^2-x^2} = \sqrt{a^2-a^2\sin^2\theta} = \sqrt{a^2(1-\sin^2\theta)} = \sqrt{a^2\cos^2\theta} = a\cos\theta$$

(B) $\sqrt{a^2+x^2} \Rightarrow x = a\tan\theta$

$dx = a\sec^2\theta\, d\theta$

$$\sqrt{a^2+x^2} = a\sec\theta$$

(b/c $1+\tan^2 = \sec^2$)

(c) $\sqrt{x^2-a^2} \Rightarrow x = a\sec\theta$

$dx = a\sec\theta\tan\theta\, d\theta$

$$\sqrt{x^2-a^2} = a\tan\theta$$

<u>Ex</u>

$$\int \frac{3}{\sqrt{x^2-9}}\, dx$$

(c) $x = 3\sec\theta$

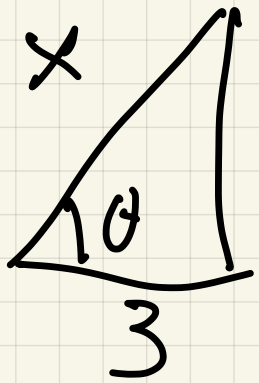
$dx = 3\sec\theta\tan\theta\, d\theta$

$$\sqrt{x^2-9} = 3\tan\theta$$

$$\int \frac{3\cdot 3\cdot \sec\theta}{3\tan\theta}\tan\theta\, d\theta$$

$$= \int 3\sec\theta\, d\theta =$$

$$3\ln|\sec\theta + \tan\theta| + C$$



$y = \sqrt{x^2-9}$

$x = 3\sec\theta$

$\frac{x}{3} = \sec\theta$

$\boxed{\frac{3}{x} = \cos\theta}$

$9 + y^2 = x^2 \Rightarrow y = \sqrt{x^2-9}$

$$3\ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right| + C$$

$$3\ln\left|\frac{x+\sqrt{x^2-9}}{3}\right| + C$$

$$3\ln|x+\sqrt{x^2-9}| - 3\ln 3 + C$$

$$3\ln|x+\sqrt{x^2-9}| + C$$

Check:

$$\frac{d}{dx}\left(3\ln\left|x+\sqrt{x^2-9}\right|\right)=$$

chain

$$3\cdot\frac{1}{x+\sqrt{x^2-9}}\cdot\left(1+\frac{x}{\sqrt{x^2-9}}\right)=$$

$$3\,\frac{1}{\cancel{x+\sqrt{x^2-9}}}\,\frac{\cancel{\sqrt{x^2-9}+x}}{\sqrt{x^2-9}}$$

$$\frac{3}{\sqrt{x^2-9}}\ \checkmark$$

Ex2 $\displaystyle\int\frac{x^3}{\sqrt{x^2+4}}\,dx$ $\quad\boxed{x=2\tan\theta}$

$dx=2\sec^2\theta\,d\theta$

$\sqrt{x^2+4}=\sqrt{4\sec^2\theta}=$

$2\sec\theta$

$$\int\frac{8\tan^3\theta}{\cancel{2}\cancel{\sec\theta}}\cdot\cancel{2}\sec^{\cancel{2}}\theta\,d\theta$$

$$= \int 8\tan^3\theta \sec\theta \, d\theta$$

$$\int 8 \underbrace{\tan^2\theta}\ \underbrace{\sec\theta \tan\theta}\, d\theta$$

$$\int 8(\sec^2\theta - 1)\sec\theta \tan\theta \, d\theta$$

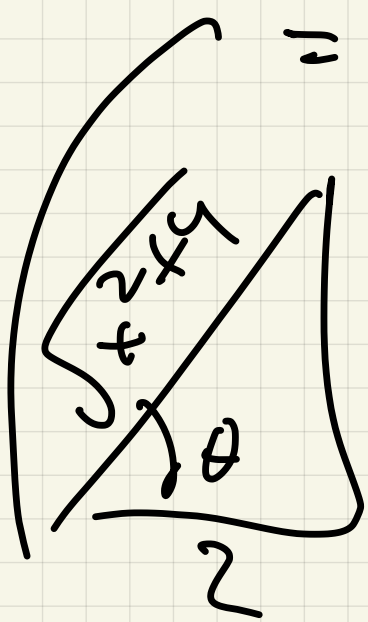
$$\boxed{u = \sec\theta}$$

$$du = \sec\theta \tan\theta \, d\theta$$

$$\int 8(u^2 - 1)\, du$$

$$\int 8u^2 - 8 = \frac{8}{3}u^3 - 8u + C$$

$$= \frac{8}{3}\sec^3\theta - 8\sec\theta + C$$



$$x = 2\tan\theta$$

$$\frac{x}{2} = \tan\theta$$

$$\sec\theta = \frac{\sqrt{x^2+4}}{2}$$

$$= \frac{8}{3}\left(\frac{\sqrt{x^2+4}}{2}\right)^3 - 8\left(\frac{\sqrt{x^2+4}}{2}\right) + C$$

$$= \frac{(x^2+4)^{3/2}}{3} - 4\sqrt{x^2+4} + C$$

<u>Ex3</u> $\int x^3 (25-x^2)^{3/2}\,dx$

$$= \int x^3\left(\sqrt{25-x^2}\right)^3 dx$$

$$\boxed{x = 5\sin\theta}$$

$$\boxed{dx = 5\cos\theta\, d\theta}$$

$$\sqrt{25-x^2} = 5\cos\theta$$

$$\int 5^3 \underline{\sin^3\theta} \cdot 5^3 \underline{\cos^3\theta} \cdot 5 \underline{\cos\theta\, d\theta}$$

$$\parallel$$

$$5^7 \int \underbrace{\sin^3\theta}_{\sin^2} \cos^4\theta \, d\theta$$

$$5^7 \int \left(1-\cos^2\theta\right) \cos^4\theta \, \sin\theta \, d\theta$$

$$\boxed{u = \cos\theta}$$

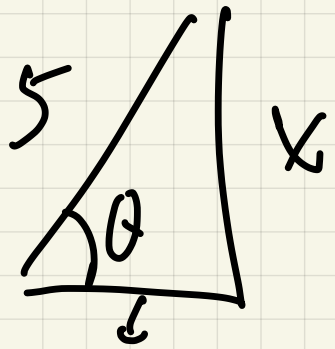
$$du = -\sin\theta \, d\theta$$

$$-5^7 \int (1-u^2)\, u^4 \, du =$$

$$-5^7 \int u^4 - u^6 \, du =$$

$$-5^7 \left(\frac{u^5}{5} - \frac{u^7}{7} \right) + C$$

$$-5^7 \left(\frac{\cos^5\theta}{5} - \frac{\cos^7\theta}{7} \right) + C$$



$$x = 5\sin\theta$$

$$\frac{x}{5} = \sin\theta$$

$\sqrt{25-x^2}$

$$\cos\theta = \frac{\sqrt{25-x^2}}{5}$$

$$-5^7\left(\frac{(25-x^2)^{5/2}}{5\cdot 5^5} - \frac{(25-x^2)^{7/2}}{7\cdot 5^7}\right)$$

$$-5(25-x^2)^{5/2} + \frac{(25-x^2)^{7/2}}{7} + C$$