2/24/ Calc 2

trig integrals

Last time

(1) Trig substitute to get of trig

(2) $\int \sin^{n} x \cos^{m} x \, dx$

n odd

$u = \cos x$

m odd

$u = \sin x$

via $\underline{\sin^2 x + \cos^2 x = 1}$

Ex1 $\int \cos^{30} 5x \, \sin^3 5x \, dx$

$=$

$\int \underbrace{\cos^{30} 5x}_{} \underbrace{(1 - \cos^2 5x)}_{} \underbrace{\sin 5x \, dx}_{}$

$u = \cos 5x$

$du = -5 \sin 5x \, dx$

$-\frac{1}{5} du = \sin 5x \, dx$

$u^{30} - u^{32}$

$$-\frac{1}{5}\int u^{30}(1-u^2)\,du =$$

$$-\frac{1}{5}\left(\frac{u^{31}}{31}-\frac{u^{33}}{33}\right)+C$$

$$-\frac{\cos^{31}5x}{155}+\frac{\cos^{33}5x}{165}+C$$

<u>Ex 2</u> (a) $\int \sin^2 7x\,dx$

Use // $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$$\left(\cos^2\theta = \frac{1+\cos 2\theta}{2}\right)$$

$$\int \frac{1-\cos 14x}{2} =$$

$$\int \frac{1}{2}-\frac{1}{2}\cos 14x\,dx =$$

$$\frac{1}{2}x-\frac{1}{28}\sin 14x + C$$

Check: $\quad \sin 2\theta = 2\sin\theta\cos\theta$

$$\left(\frac{1}{2}x - \frac{2}{28}\sin 7x\cos 7x\right)'$$

$$\frac{1}{2} - \frac{1}{2}\cos^2 7x + \frac{1}{2}\sin^2 7x$$

$$\frac{1}{2}\sin^2 7x \quad + \quad \sin^2 7x \checkmark$$

(b) $$\int_0^{\pi} \sin^4 t \, dt =$$

$$\int_0^{\pi} (\sin^2 t)(\sin^2 t)\, dt$$

$$\int_0^{\pi} \left(\frac{1-\cos 2t}{2}\right)\left(\frac{1-\cos 2t}{2}\right) dt$$

$$\frac{1}{4}\int_0^{\pi} 1 - 2\cos 2t + \cos^2 2t$$

$$\parallel$$

$$\frac{1}{4}\int 1 - 2\cos 2t \quad \frac{1+\cos 4t}{2}\,dt$$

$$\frac{1}{4}\int \underline{\underline{1}} - 2\cos 2t + \frac{1}{2} + \frac{\cos 4t}{2}$$

$$\frac{1}{4}\left(t - \sin 2t + \frac{1}{2}t + \frac{\sin 4t}{8} \right)\Big|_0^{\pi}$$

$$= \frac{1}{4}\left(\pi + \frac{\pi}{2} \right) = \frac{3\pi}{8}$$

(B) Integrals and sec x/tan x

$$\boxed{\sec^2 x - 1 = \tan^2 x} \leftarrow$$

(a) $$\int \frac{\boxed{\tan^2 x}}{\sec x}\,dx = \int \frac{\sec^2 x - 1}{\sec x}\,dx$$

$$= \int \frac{\sec^2 x}{\sec x} - \frac{1}{\sec x}\,dx$$

$$= \int \sec x - \cos x\,dx$$

$\ln|\sec x+\tan x| - \sin x + C$

(b) $\int \frac{\tan^2 x}{\sec^5 x}\,dx$ $\tan x = \frac{\sin}{\cos}$

$\sec = \frac{1}{\cos}$

$$\int \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^5 x}} = \int \sin^2 x\, \underline{\cos^3 x}\, dx$$

$u = \sin x$

$$= \int \sin^2 x\,(\underline{1-\sin^2 x})\cos x\, dx$$

$$\int u^2(1-u^2)\,du$$

$$= \int u^2 - u^4\, du =$$

$$\frac{1}{3}u^3 - \frac{1}{5}u^5 + C$$

$$\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$$

(c) $\int \sec^4 x \, dx =$

$\int \underline{\sec^2 x} \sec^2 x \, dx$

$\int (1+\tan^2 x) \underline{\sec^2 x \, dx}$

$\quad \| \quad u = \tan x$

$\quad \| \quad du = \sec^2 x \, dx$

$\int (1+u^2) \, du$

$u + \frac{1}{3} u^3 + C$

$\tan x + \frac{1}{3} \tan^3 x + C$

<u>In general</u> $\int \tan^m x \sec^n x \, dx$

if <u>n even</u>, then $u = \tan x$

(d) $\int \tan^5 x \sec^2 x \, dx$

$u = \tan x \quad \|$

$$\int u^5\,du = \frac{1}{6}u^6 + C$$

$$= \frac{1}{6}\tan^6 x + C$$

(e) $\int \underline{\tan^4 x\,dx}$

$$\int \underbrace{(\sec^2 x - 1)}_{\tan^2 x} \tan^2 x\,dx =$$

$$\int \tan^2 x \sec^2 x - \underline{\tan^2 x}\,dx$$

$$\int \tan^2 x \sec^2 x - (\sec^2 x - 1)\,dx$$

$$= \int \tan^2 x \sec^2 x - \sec^2 x + 1\,dx$$

$u = \tan x$

$$\int u^2\,du - \int du + \int dx$$

$$\frac{1}{3}u^3 - u + 1 + C$$

$$\frac{1}{3}\tan^3 x - \tan x + 1 + C$$

(f) $\int \tan^3 x \sec^5 x \, dx$

$= \int \underline{\tan^2 x} \sec^4 x \; \sec x \tan x \, dx$

$u = \sec x$

$\int (\sec^2 x - 1) \sec^4 x \; \underbrace{\sec x \tan x \, dx}_{du}$

$\int (u^2 - 1) \; u^4$

$\int u^6 - u^4 \, du =$

$\frac{1}{7} u^7 - \frac{1}{5} u^5 + C$

$\frac{1}{7} \sec^7 x - \frac{1}{5} \sec^5 x + C$

(g) $\int \tan^5 x \sec^8 x \, dx$

both methods above work:

$\int \tan^4 x \sec^7 x \; \underline{\sec x \tan x \, dx}$

$\int (1 - \sec^2 x)(1 - \sec^2 x) \sec^7 x \sec x \tan x \, dx$

$u = \sec x$

$$\int \underline{(1-u^2)^2}\, u^7\, du =$$

$$\int (1-2u^2+u^4)\, u^7\, du =$$

$$\int u^7 - 2u^9 + u^{11}\, du =$$

$$\frac{u^8}{8} - \frac{2}{10}u^{10} - \frac{u^{12}}{12} + C$$

$$\frac{\sec^8 x}{8} - \frac{1}{5}\sec^{10} x - \frac{\sec^{12} x}{12} + C$$

<u>In general</u> $\int \sec^n x \tan^m x\, dx$,

m odd, $n > 0$, $u = \sec x$

$$(1 + \tan^2 x = \underset{\text{wole}}{\sec^2 x})$$

(h) $\int \tan x \sec^{90} x\, dx$

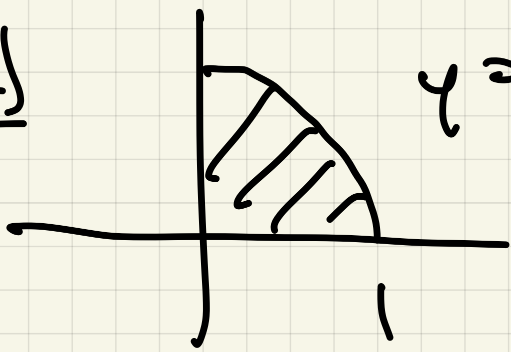
$$\int \sec^{89} x\, \sec x \tan x\, dx$$

$u = \sec x$

$$\int u^{89}\,du = \frac{u^{90}}{90} + C$$

$$\frac{\sec^{90} x}{90} + C$$

Ex



$y = \sqrt{1-x^2}$

$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}$$

second idea:

trick: $\boxed{x = \sin\theta}$ $\boxed{\theta = \arcsin x}$

$dx = \cos\theta\,d\theta$

$\theta = \pi/2$

$\theta = 0$

$$\int \sqrt{1-x^2}\,dx = \int \sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta$$

$$\int \sqrt{\cos^2\theta}\,\cos\theta\,d\theta = \int \cos\theta\cos\theta$$

$$\int_0^{\pi/2} \cos^2\theta\,d\theta =$$

$$\int_0^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2}\,dx = \frac{1}{2}\theta + \frac{\sin 2\theta}{4}\Big|_0^{\pi/2}$$

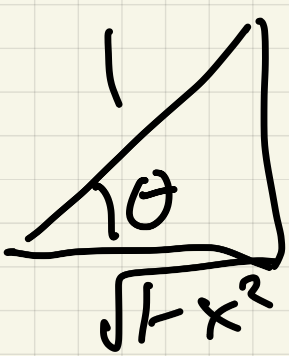
$$= \frac{\pi}{4} \checkmark$$

Indefinite integral:

$$\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C$$

$$\frac{1}{2}\theta + \frac{1}{4}\cdot 2\sin\theta\cos\theta + C$$

$$\parallel$$



$$\frac{1}{2}\arcsin x + \frac{1}{2}x\sqrt{1-x^2} + C$$

$\theta = \arcsin x$

Ex 2 Find arc length

$$y = \frac{1}{2}x^2, \qquad 0 \le x \le 1$$

$$dy/dx = x$$

so $$L = \int_0^1 ds = \int_0^1 \sqrt{1+x^2}\,dx$$

$$x = \tan\theta$$
$$dx = \sec^2\theta\, d\theta$$

$$\int \sqrt{\underbrace{1+\tan^2\theta}_{\sec\theta}}\ \sec^2\theta\, d\theta$$

$$\int \sec\theta \sec^2\theta\, d\theta$$

$$\int \sec^3\theta\, d\theta$$

|| Int. by parts

$$\frac{1}{2}\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right)\Big|_0^{\pi/4}$$

$$\frac{\sqrt{2} + \ln(\sqrt{2}+1)}{2}$$