2/23/Calc 2

Exam 1

avg 80%

med 83%

150 — 1

135 — 6

120 — 9

105 — 5

4

#1 Good

$\boxed{\#2}$

y-axis Washers

$$\int_1^3 \pi (y^3)^2 \, dy =$$

$$\pi \frac{y^7}{7} \Big|_1^3 = \pi \left(\frac{3^7 - 1}{7} \right)$$

(b) $y = -1$ Shells

$$\int 2\pi y (y^3) \, dy$$

$$\int_1^3 2\pi (y+1)(y^3)\,dy$$

$\boxed{3}$

<u>y-axis</u> : <u>shells</u> :

(a) $\displaystyle\int_0^{\pi} 2\pi x \sin x \, dx$

(b) $\displaystyle\int_0^{\pi} 2\pi (5-x)\sin x \, dx$

(c) $\displaystyle\int_0^{\pi} \pi (\text{out}^2 - \text{in}^2)\, dx$

$$\int_0^{\pi} \pi\left((1+\sin x)^2 - 1^2\right) dx$$

$y = -1$

$\boxed{5}$

$\rho = 40$

$$r^2 + y^2 = 10^2 = 100$$

$$r = \sqrt{100 - y^2}$$

$$A(y) = \pi(100 - y^2)$$

$$D(y) = y + 20$$

$$W = \int_0^7 40 \cdot \pi(100 - y^2)(y + 20)\,dy$$

<u>Last time</u>:

$$\int u\,dv = uv - \int v\,du$$

std	$dv = dx$	fold back
$\int x\sin x$	$\int \ln x\,dx$ $\int \arctan x\,dx$	$\int e^x \cos x\,dx$

§8.2 Trig integrals

$\int \cos x\,dx$

$\int \sin x\,dx$

$\int \sec^2 x\,dx$

$\int \sec x \tan x\,dx$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\int \tan x\,dx = -\ln|\cos x| + C$

$\int \frac{\sin x}{\cos x}\,dx$

Identities:

$$\boxed{\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \tan^2 x + 1 &= \sec^2 x \end{aligned}}$$

$$\boxed{\begin{aligned} \cos^2 x &= \frac{1+\cos 2x}{2} \\ \sin^2 x &= \frac{1-\cos 2x}{2} \end{aligned}}$$

(A) Integrals involving $\sin x$ / $\cos x$

(a) $\displaystyle\int \underbrace{\sin^6 x}_{u^6} \underbrace{\cos x\, dx}_{du} =$

$u = \sin x$

$du = \cos x\, dx$

$$\int u^6\, du = \frac{u^7}{7} + C$$

$$= \frac{\sin^7 x}{7} + C$$

(b) $\displaystyle\int \frac{\boxed{\cos^2 x}\,\boxed{-\sin x}\, dx}{\boxed{\cos^2 x + 1}} =$

$u = \cos x \qquad du = -\sin x\, dx$

$$\int \frac{-u^2\, du}{(u^2+1)} =$$

$$-\int \frac{u^2}{u^2+1}\,du = -\int \underbrace{\frac{u^2+1}{u^2+1}}_{=1} - \frac{1}{u^2+1} =$$

$$-\int 1\,du + \int \frac{du}{u^2+1}$$

$$-u + \arctan u + C$$

$$-\cos x + \arctan(\cos x) + C$$

(c) $\int \cos x\, e^{\sin x}\,dx$

$u = \sin x$

$du = \cos x\,dx$

$$= \int e^u\,du =$$

$$e^u + C = e^{\sin x} + C$$

Powers of sin/cos

(d) $\int \sin^4 x \underbrace{\cos^3 x}\,dx$

// trick $\cos^3 x \cos x$

$= (1-\sin^2 x)\cos x$

$$\int \sin^4 x (1-\sin^2 x) \underbrace{\cos x \, dx}$$

$u = \sin x \quad du = \cos x \, dx$

$$\int u^4 (1-u^2) \, du =$$

$$\int u^4 - u^6 \, du$$

$$\frac{u^5}{5} - \frac{u^7}{7} + C$$

$$\frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$$

(e) $\int \underbrace{\sin^3 x} \cos^8 x \, dx$

$\sin^2 x = 1 - \cos^2 x$

$$\int (1-\cos^2 x) \cos^8 x \sin x \, dx$$

$u = \cos x \quad du = -\sin x \, dx$

$$-\int (1-u^2)\,u^8\,du =$$

$$\int u^{10} - u^8\,du =$$

$$\frac{u^{11}}{11} - \frac{u^9}{9} + C \quad \leftarrow u = \cos x$$

(f) $\int \sin x \cos^5 x\,dx$

<u>Easy:</u> $u = \cos x$

$du = -\sin x$

<u>hard</u> $\cos^5 x = (1-\sin^2 x)(1-\sin^2 x)\cos x$

$$\int \sin x(1-\sin^2 x)(1-\sin^2 x)\cos x\,dx$$

$u = \sin x$

<u>Conclusion:</u>

$$\int \sin^n x \cos^m x\,dx$$

<u>n odd</u>: $u = \cos x$

<u>m odd</u>:

$u = \sin x$

use trig identity to help

Ex $\int \cos^5 x \, dx$

$\int \cos^2 x \cos^2 x \cos x \, dx$

$u = \sin x$

$\int (1 - \sin^2 x)(1 - \sin^2 x) \cos x \, dx$

$\int (1 - u^2)(1 - u^2) \, du$

$\int 1 - 2u^2 + u^4 \, du$

$u - \frac{2}{3}u^3 + \frac{u^5}{5} + C$

$\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$