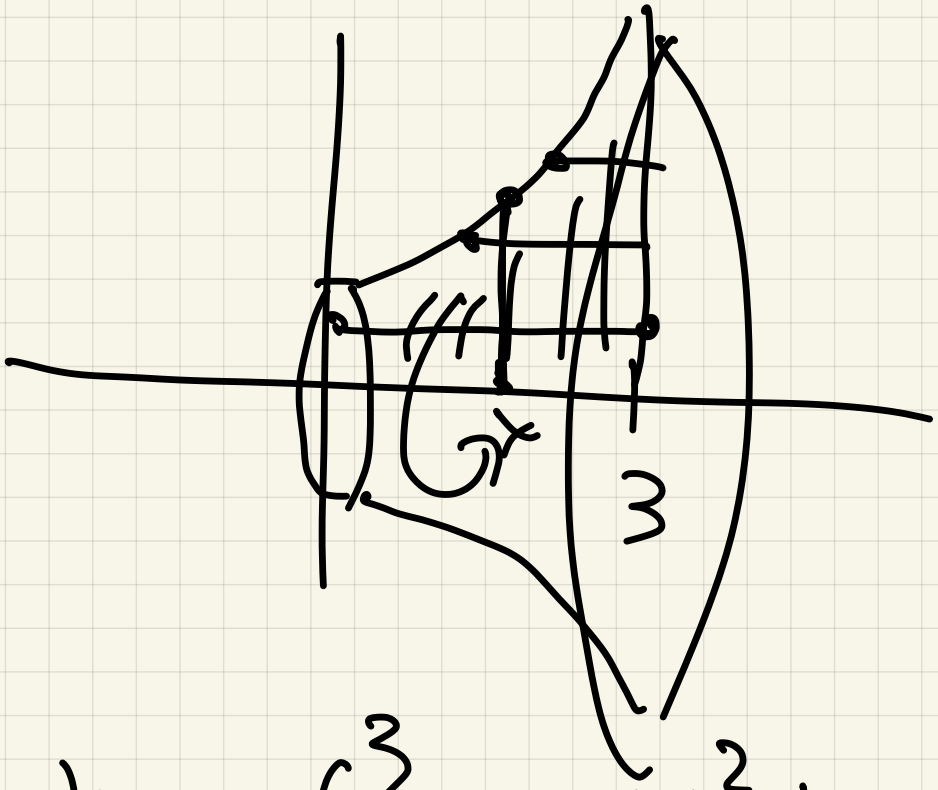
2/2/Calc2 **Quiz 3** avg ~ 78
med = 80

#1 $y = e^x$, $y = 0$, $x = 0$, $x = 3$



x-axis

disk: $\int_0^3 \pi (e^x)^2 \, dx =$

$\int_0^3 \pi e^{2x} \, dx = \frac{\pi e^{2x}}{2} \Big|_0^3 =$

$\frac{\pi(e^6 - 1)}{2}$

e^0

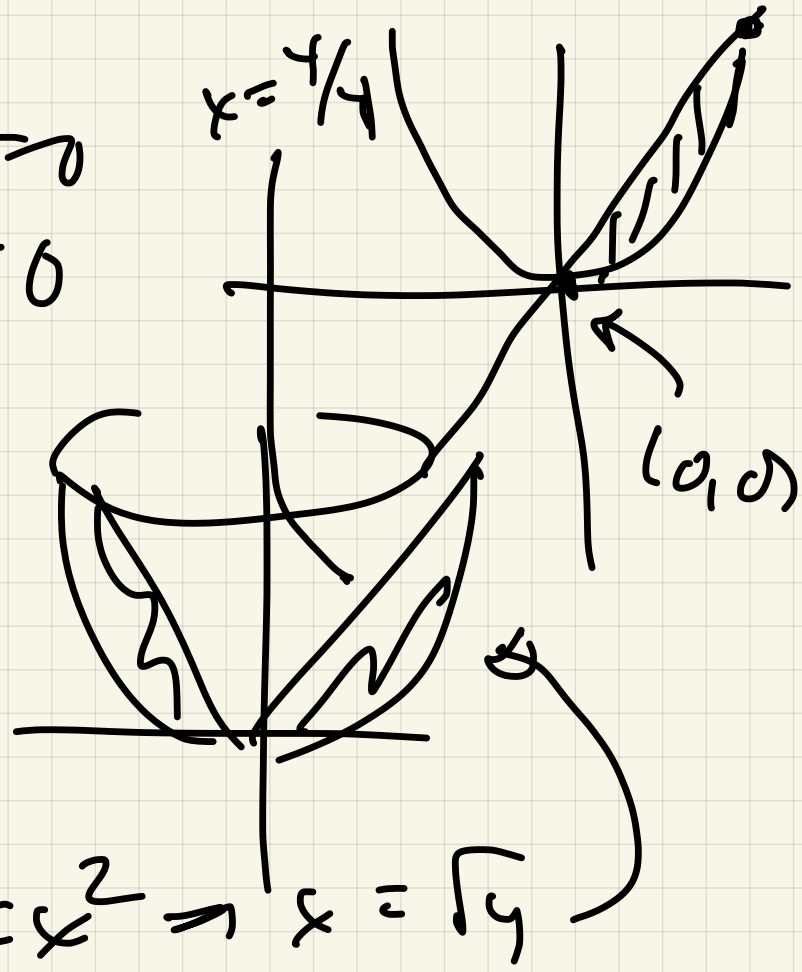
#2 $y = 4x$, $y = x^2$ (4,16)

$4x = x^2$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0, 4$



2 ways

$y = 4x$

$\frac{y}{4} = x$

$y = x^2 \Rightarrow x = \sqrt{y}$

$V = \int \pi(\text{outer}^2 - \text{inner}^2)$

$= \int_0^{16} \pi\left(\left(\sqrt{y}\right)^2 - \left(\frac{y}{4}\right)^2\right) dy =$

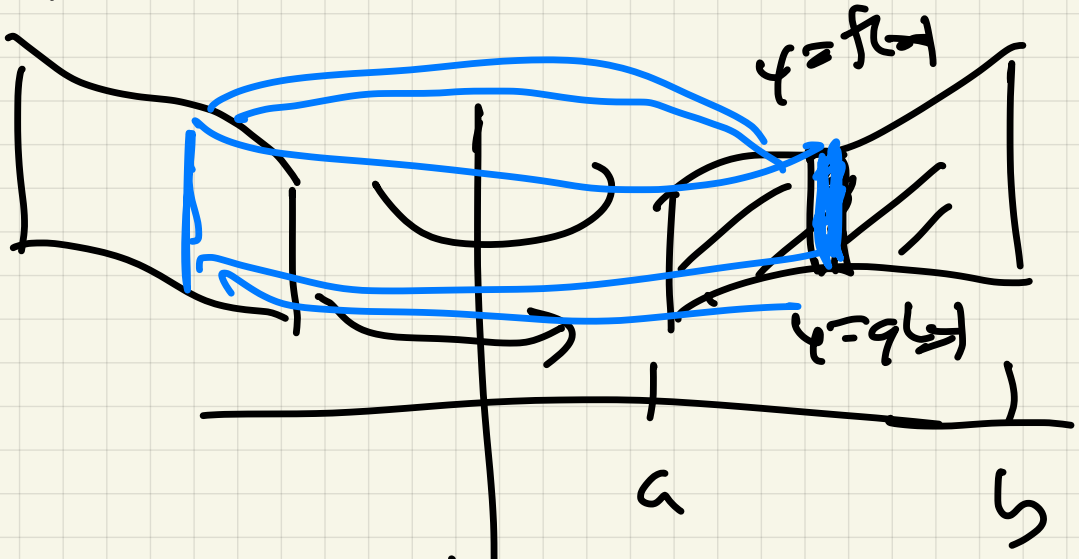
$\int_0^{16} \pi\left(y - \frac{y^2}{16}\right) dy = \pi\left(\frac{y^2}{2} - \frac{y^3}{16 \cdot 3}\right)\Big|_0^{16} =$

$$\pi\left(\frac{16^2}{2} - \frac{16^2}{16 \cdot 3}\right) =$$

$$\pi\left(\frac{256}{2} - \frac{256}{3}\right) = \frac{256\pi}{6} =$$

$$\frac{128\pi}{3}.$$

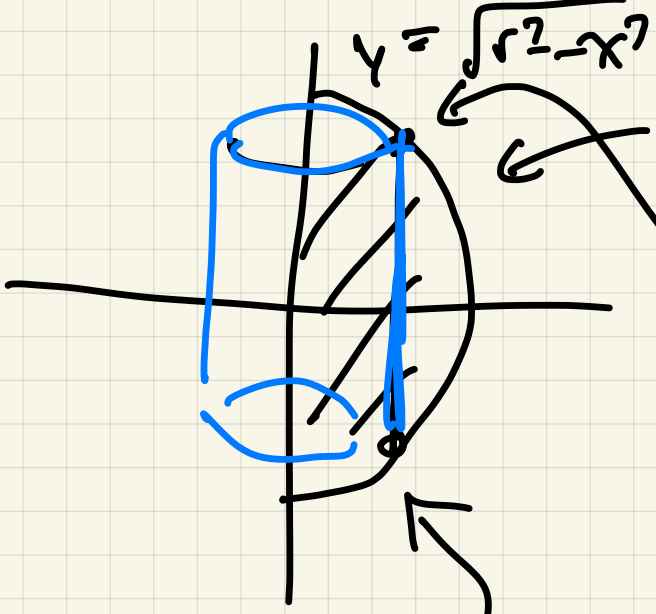
Last time: Shell method:



$$V = \int_a^b 2\pi x \cdot \underbrace{(f(x) - g(x))}_{ht} dx$$

radius

Ex1 Sphere of radius r



$x^2 + y^2 = r^2$

revolve about y-axis

$y = \pm\sqrt{r^2 - x^2}$

$y = -\sqrt{r^2 - x^2}$

$$V = \int 2\pi \cdot x \cdot (\text{height}) \, dx$$

x

$\sqrt{r^2 - x^2} + \sqrt{r^2 - x^2}$

$2\sqrt{r^2 - x^2}$

$$\int_0^r 2\pi x 2\sqrt{r^2 - x^2}\, dx =$$

$$4\pi \int_0^r x\sqrt{r^2 - x^2}\, dx =$$

$u = r^2 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$u^{1/2}$

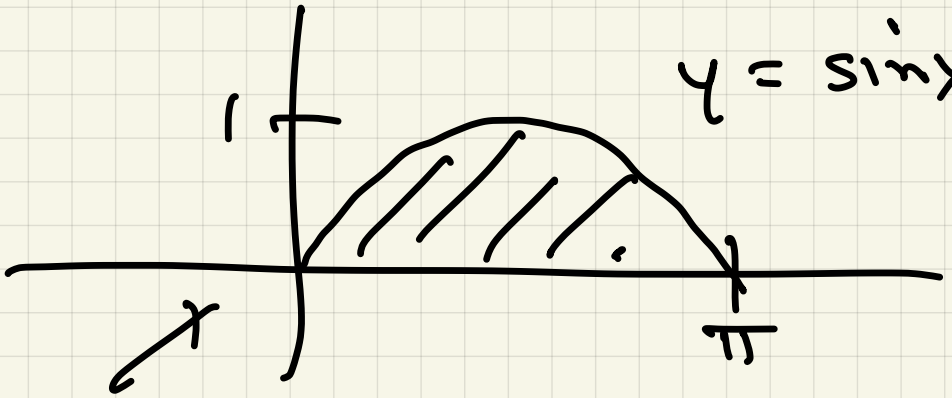
$$4\pi\left(-\frac{1}{2}\right)\int_{r^2}^{0} \sqrt{u}\, du =$$

$$-2\pi \frac{2}{3} u^{3/2} \Big|_{r^2}^{0} =$$

$$0 - \left(-2\pi \cdot \frac{2}{3} (r^2)^{3/2}\right)$$

$$= \frac{4\pi}{3} r^3$$

Ex1 SET UP integrals for volume of revolution of R



boundary $y = \sin x$, $y = 0$

$0 \le x \le \pi$

a) x-axis

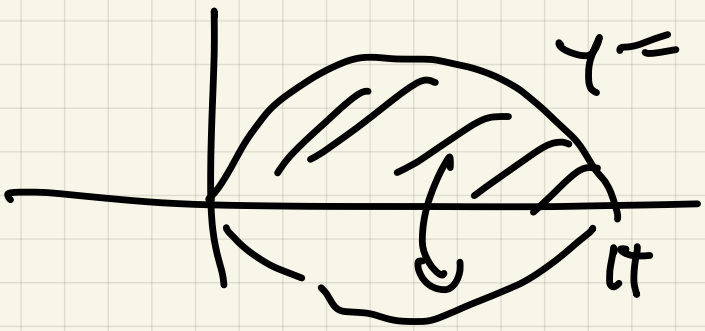
b) $y = 3$

c) $y = -1$

d) y-axis

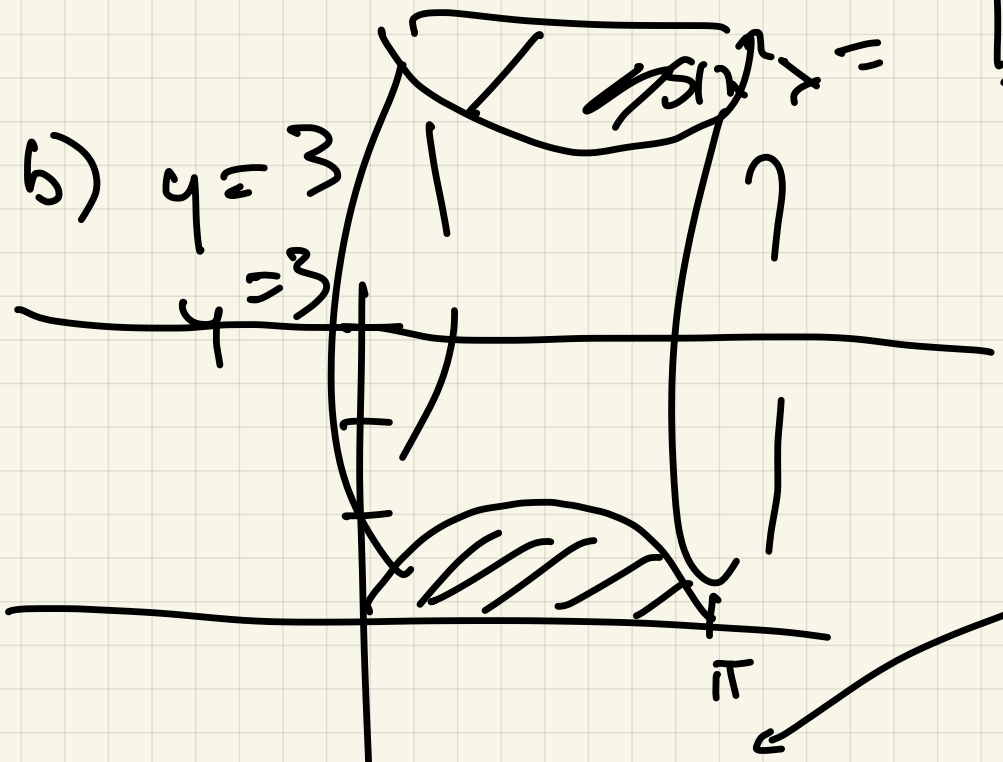
e) $x = -\pi$

f) $x = \pi$



a) <u>disk</u>: $\int_0^{\pi} \pi(\sin x)^2 dx = \frac{\pi^2}{2}$

($\sin^2 x = \frac{1-\cos 2x}{2}$

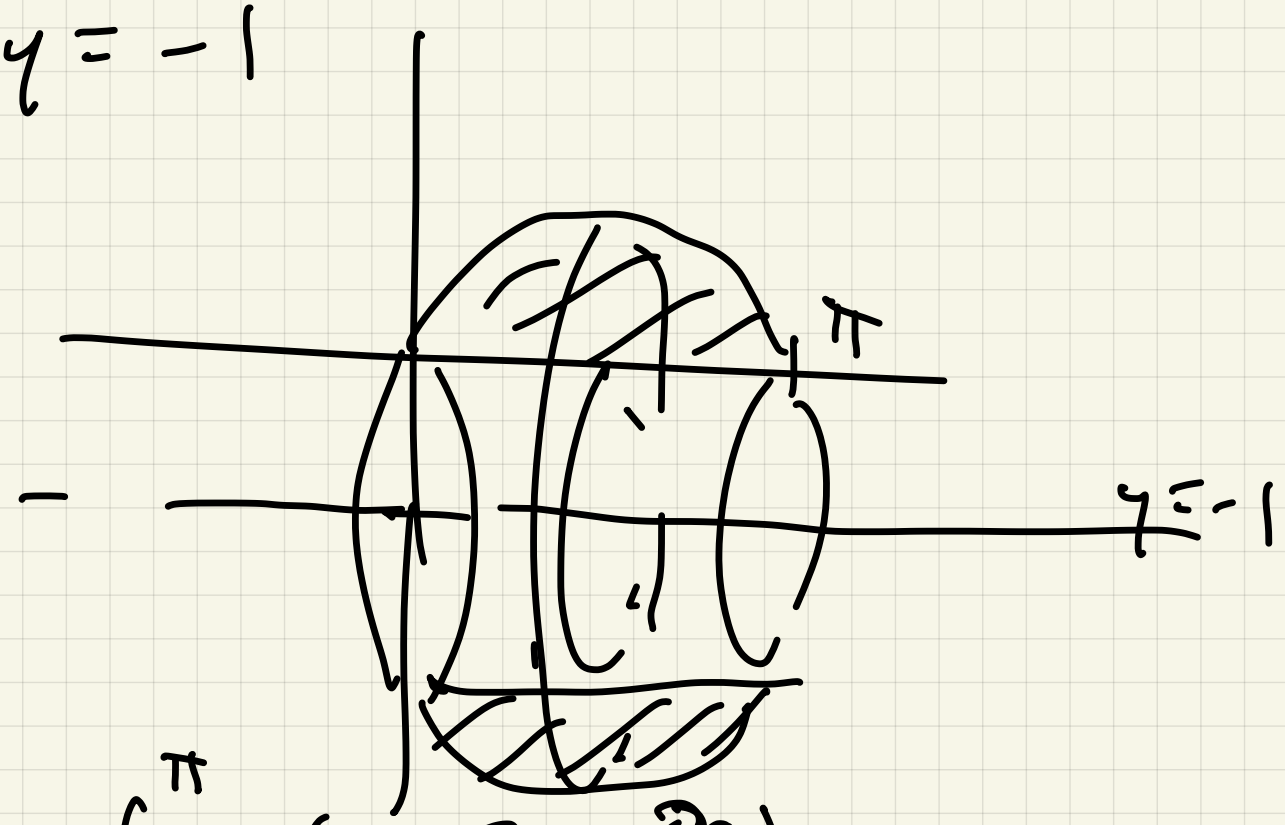


<u>washers</u>:

$$\int_0^{\pi} \pi(\text{out}^2 - \text{in}^2)\, dx =$$

$$= \int_0^{\pi} \pi\left(3^2 - (3-\sin x)^2\right) dx$$

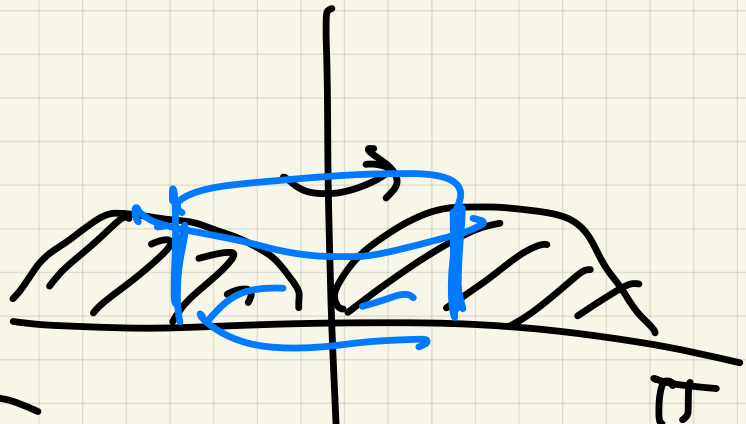
(c) $y = -1$



$$V = \int_0^{\pi} \pi\left(\quad ^2 - 1^2 \right) dx =$$

$$\int_0^{\pi} \pi\left((\sin x + 1)^2 - 1^2 \right) dx$$

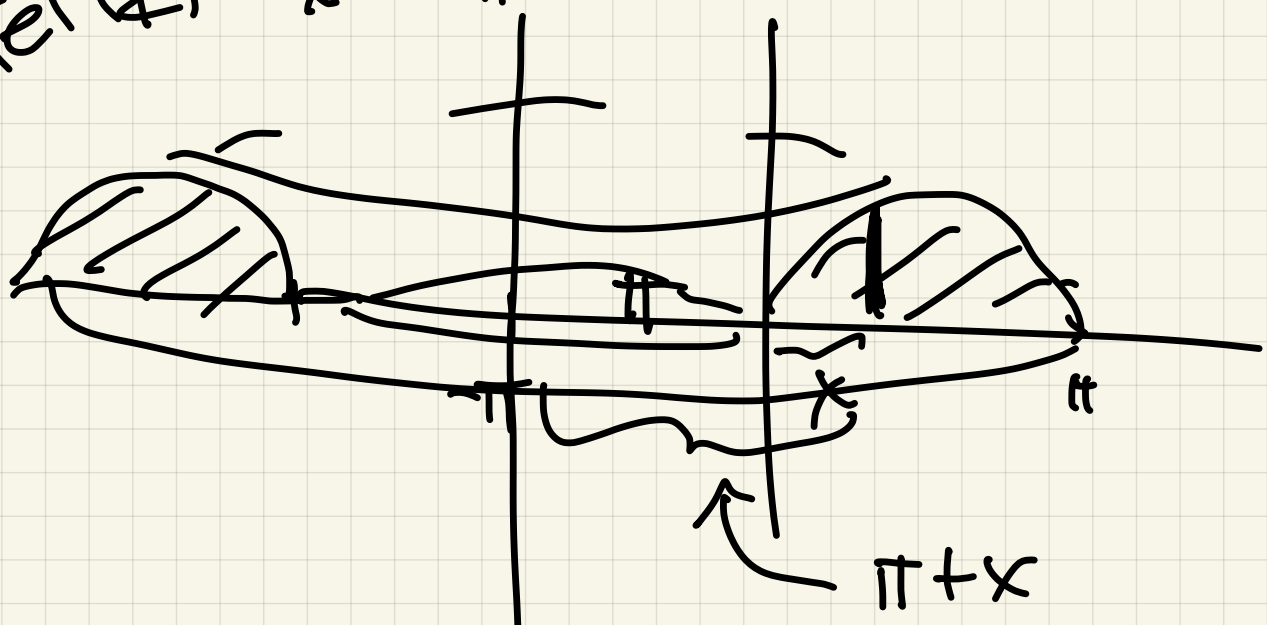
(d) $x = \arcsin y$



Shells: $\int_0^{\pi} 2\pi(\text{rad})(\text{ht})$

$$\int_0^{\pi} 2\pi x \sin x \, dx = 2\pi^2$$

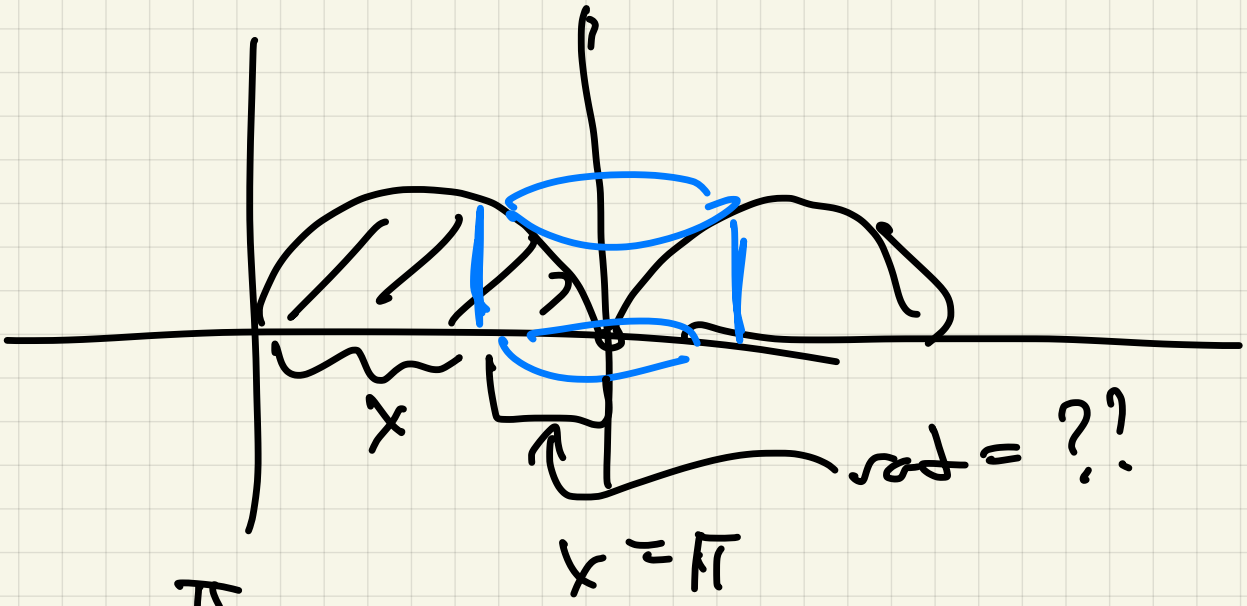
(예제) $x = -\pi$



$$V = \int_0^{\pi} 2\pi \cdot \text{rad} \cdot \text{ht} \, dx$$

$$= \int_0^{\pi} 2\pi \underbrace{(\pi + x)}_{\text{rad.}} \sin x \, dx$$

(참) $x = \pi$



$$V = \int_0^{\pi} 2\pi \ \text{rad} \ \text{ht} \ dx$$

$$\int_0^{\pi} 2\pi \ (\pi - x) \ \sin x \, dx$$