Exam 3 (pink)

$\boxed{1}$ $a_n = \dfrac{72-4n^2}{n^2} = \dfrac{72}{n^2} - 4$

(a) $a_1 = \frac{72}{1} - 4 = 68$, $a_2 = \frac{72}{4} - 4 = 14$, $a_3 = \frac{72}{9} - 4 = 4$

(b) $\lim_{n\to\infty} \frac{72}{n^2} - 4 = -4$

(c) a_n is monotone: $\frac{d}{dn}\left(\frac{72}{n^2} - 4\right) = \frac{-144}{n^3} < 0$

$\Rightarrow a_n$ decreasing

(d) $-4 \leq a_n \leq 68$

$\boxed{2}$ (a) $S_1 = 36(-2/3) = -24$, $S_2 = 36(-2/3) + (36)(-2/3)^2$

$= -24 + 16 = -8$

(b) series geometric, $r = -2/3$,

$|r| = 2/3 < 1 \Rightarrow$ converges

(c) $\dfrac{a}{1-r} = \dfrac{-24}{1-(-2/3)} = \dfrac{-24}{5/3} = \dfrac{-72}{5}$

$\boxed{3}$ (a) $\lim_{n\to\infty} \left| \dfrac{\frac{(n+1)!}{7^{n+1}}}{\frac{n!}{7^n}} \right| = \lim_{n\to\infty} \dfrac{(n+1)!}{n!} \cdot \dfrac{7^n}{7^{n+1}} =$

$\lim_{n\to\infty} \dfrac{n+1}{7} = +\infty > 1 \Rightarrow$ divergent

(b) $\sum \frac{3k}{k^3+7} \sim \frac{1}{k^2}$ convergent p-series, $p = 2 > 1$

$$\lim_{k\to\infty} \frac{\frac{3k}{k^3+7}}{\frac{1}{k^2}} = \lim_{k\to\infty} \frac{3k^3}{k^3+7} = 3 \neq 0,$$

$\therefore \sum \frac{3k}{k^3+7}$ converges by LCT

(c) $\lim_{n\to\infty} \sqrt[n]{\left(\frac{2n+3}{3n+2}\right)^n} = \lim_{n\to\infty} \left(\frac{2n+3}{3n+2}\right) = \frac{2}{3} < 1$

$\therefore$ Converges by root test

(d) $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k+4} \sim \frac{\sqrt{k}}{k} = \frac{1}{\sqrt{k}}$ divergent p-series, $p = 1/2 <1$

$$\lim_{k\to\infty} \left| \frac{\frac{\sqrt{k}}{k+4}}{\frac{1}{\sqrt{k}}} \right| = \lim \frac{k}{k+4} = 1, \text{ so}$$

$\sum \frac{\sqrt{k}}{k+4}$ diverges by LCT

(4) $f(x) = \frac{1}{x(\ln x)^4}$

cont ✓ $x > 0$

positive $x > 1$

decre $x > 1$

because x, $\ln x$ grow $\Rightarrow$

$\frac{1}{x(\ln x)^4}$ shrinks, so Int test applies

$$\int_2^\infty \frac{1}{x(\ln x)^4} = \lim_{b\to\infty} -\frac{1}{3(\ln x)^3}\Big|_2^b =$$

$u = \ln x \Rightarrow \int \frac{1}{u^4}du = -\frac{1}{3u^3}$ $\quad \lim_{b\to\infty} -\frac{1}{3\ln^3 b} + \frac{1}{3\ln^3 2} =$

$\frac{1}{2\ln^4 2}$ converges $\therefore \sum \frac{1}{k\ln^4 k}$ conv by integral test.

$\boxed{5}$ (a) $\displaystyle\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+2}(x-4)^{n+1}}{(n+1)\cdot 6^{n+1}}}{\frac{(-1)^{n+1}(x-4)^n}{n\cdot 6^n}}\right| = \lim_{n\to\infty}\frac{n}{n+1}\frac{|x-4|}{6}$

$= \frac{|x-4|}{6} < 1 \Rightarrow -6 < x-4 < 6 \Rightarrow -2 < x < 10$

$\underline{x=-2}$ $\quad \sum \frac{(-1)^{2n+1}}{k} = \sum \frac{-1}{k}$ divergent p-series, $p=1$

$\underline{x=10}$ $\quad \sum \frac{(-1)^{n+1}}{n}$ AST $\Rightarrow$ convergent, n

I.O.C. $= (-2, 10]$

(b) $R = 6$

(c) $\displaystyle\int f = C + \sum_{k=0}^{\infty} \frac{(-1)^{k+1}(x-4)^{k+1}}{k\cdot(k+1)\cdot 6^k}$

$\boxed{6}$ (a) <u>abs. conv</u> : LCT w/ $\sum \frac{1}{k^3}$

(b) <u>cond convergent</u> conv by AST, abs divergent by

LCT w/ $\sum \frac{1}{\sqrt{k}}$

(c) <u>divergent</u> by n^{th} term ~~test~~

(d) converges by AST, abs diverges by LCT w/ $\sum \frac{1}{k}$ $\therefore$ <u>cond convergent</u>

(e) <u>Abs convergent</u> by Ratio test.