Exam 3 (blue)

$\boxed{1}$ (a) $a_1 = \frac{3-108}{1} = -105$, $a_2 = \frac{12-108}{4} = \frac{-96}{4} = -24$

$a_3 = \frac{27-108}{9} = -\frac{81}{9} = -9$

(b) $a_n = \frac{3n^2-108}{n^2} = 3 - \frac{108}{n^2}$, so

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} 3 - \frac{108}{n^2} = 3$

(c) a_n is monotone: $a_n = 3 - \frac{108}{n^2}$

$\frac{d}{dn}(a_n) = \frac{216}{n^3} > 0 \Rightarrow a_n$ increasing

(d) $-105 \le a_n \le 3$

$\boxed{2}$ (a) $s_1 = 32\left(\frac{-3}{4}\right) = -24$, a

$s_2 = 32\left(\frac{-3}{4}\right) + 32\left(-\frac{3}{4}\right)^2 = -24 + 18 = -6$

(b) Series is geometric, $r = -3/4$,

$\therefore |r| = |3/4| < 1 \Rightarrow$ convergent

(c) $s = \frac{a}{1-r} = \frac{-24}{1-(-3/4)} = \frac{-24}{(7/4)} = \frac{-96}{7}$

$\boxed{3}$ (a) $\sum_{k=1}^{\infty} \frac{3k}{k^2+2} \sim \frac{1}{k}$, $\sum \frac{1}{k}$ divergent p-series ($p=1$)

$$\lim_{k\to\infty} \frac{\frac{3k}{k^2+1}}{\frac{1}{k}} = \lim_{k\to\infty} \frac{3k^2}{k^2+1} = \lim_{k\to\infty} \frac{3}{1+1/k^2} = 3,$$

so $\sum \frac{3k}{k^2+2}$ <u>diverges by LCT</u>

(b) $\lim_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)!}}{\frac{5^n}{n!}} = \lim \frac{n!}{(n+1)!} \cdot \frac{5^{n+1}}{5^n} =$

$\lim_{n\to\infty} \frac{5}{n+1} = 0 < 1 \Rightarrow$ <u>convergent</u> by Ratio test

(c) $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2+4} < \frac{\sqrt{k}}{k^2} = \frac{1}{k^{3/2}},$ ← convergent p-series, $p = 3/2 > 1$

so $\sum \frac{\sqrt{k}}{k^2+4}$ <u>converges</u> by DCT.

(d) $\lim_{n\to\infty} \sqrt[n]{\left(\frac{4n+3}{3n+2}\right)^n} = \lim \frac{4n+3}{3n+2} = \frac{4}{3} > 1 \Rightarrow$

series <u>diverges</u> by Root test.

$\boxed{4}$ $f(x) = \frac{1}{x(\ln x)^3}$ is <u>continuous</u> ✓ $(x>0)$
<u>positive</u> $(x>1)$
decreasing $(x>1)$

b/c x, $(\ln x)$ non increasing

$\therefore$ Integral test applies:

$$\int_2^\infty \frac{1}{x(\ln x)^3}\,dx = \lim_{b\to\infty} -\frac{1}{2(\ln x)^2}\Big|_2^b =$$

$u = \ln x$, $du = \frac{1}{x}$ $\quad \int \frac{1}{u^3} = -\frac{1}{2u^2}$

$$\lim_{b\to\infty} -\frac{1}{2(\ln b)^2} + \frac{1}{2(\ln 2)^2} = \frac{1}{2(\ln 2)^2},$$

$\therefore$ series converges.

$\boxed{5}$ (a) $\displaystyle \lim_{n\to\infty}\left| \frac{\dfrac{(-1)^{n+2}(x-5)^{n+1}}{\sqrt{n+1}\;7^n}}{\dfrac{(-1)^{n+1}(x-5)^n}{\sqrt{n}\;7^{n+1}}} \right| = \lim_{n\to\infty} \sqrt{\frac{n}{n+1}}\,\frac{(x-5)}{7}$

$= \frac{|x-5|}{7}$, $\quad -1 < \frac{x-5}{7} < 1 \Rightarrow -7 < x-5 < 7$

$\Rightarrow -2 < x < 12$

$\underline{x=-2} \Rightarrow \sum \frac{(-1)^{2k+1}}{\sqrt{k}} = \sum \frac{-1}{\sqrt{k}}$ diverges p-series $p = 1/2 \le 1$

$\underline{x=12} \Rightarrow \sum \frac{(-1)^{k+1}}{\sqrt{k}}$ converges by AST

(so I.O.C. $= (-2, 12]$ $\boxed{\text{(c) see end}}$

$\boxed{6}$ (a) $\lim_{n\to\infty} (-1)^n \frac{2n}{7n+8}$ DNE, so

$\frac{2n}{7n+8} \to 2/7$

diverges by nth term test.

(b) $\sum (-1)^k \frac{k^2}{3^k}$ absolutely convergent by Ratio test

(c) $\sum (-1)^k \frac{8k}{k^3+3} \sim \frac{1}{k^2}$ absolutely convergent by LCT

(d) $\sum (-1)^k \frac{5k}{k^2+4}$ converges by AST,

but $\sum \frac{5k}{k^2+4} \sim \frac{1}{k}$ diverges $\left(\begin{matrix}\text{p-series,} \\ \text{LCT}\end{matrix}\right)$

$\therefore$ converges conditionally

(e) $\sum (-1)^k \frac{k^2}{\sqrt{k^7-5}}$ $\quad \frac{k^2}{\sqrt{k^7-5}} \sim \frac{k^2}{k^{7/2}} = \frac{1}{k^{3/2}}$

convergent p-series $p = 3/2$ $\quad \therefore$ absolutely convergent.

$\boxed{5(c)}$ $\displaystyle\int \sum_{0}^{\infty} \frac{(-1)^{k+1}(x-5)^k}{\sqrt{k}\cdot 7^k} = C + \sum_{k=0}^{\infty} \frac{(-1)^{k+1}(x-5)^{k+1}}{(k+1)\sqrt{k}\cdot 7^k}$