Exam 2 (yellow)

$\boxed{1}$ (a) $\displaystyle\int \frac{x^3}{\sqrt{16-x^2}}\,dx$ $\quad x = 4\sin\theta$

$dx = 4\cos\theta\, d\theta$

$\sqrt{16-x^2} = 4\cos\theta$

$\parallel$

$$\int \frac{(4\sin\theta)^3\, 4\cancel{\cos\theta}}{4\cancel{\cos\theta}} = 64\int \sin^3\theta\, d\theta$$

(b) $$\frac{5}{x^2(x^2-9)(x^2+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3} + \frac{D}{x+3} + \frac{Ex+F}{x^2+1} + \frac{Gx+H}{(x^2+1)^2}$$

(c) $\displaystyle\int_1^\infty \frac{dx}{x^{3/5}}$ diverges b/c $p = 3/5 < 1$

(d) $$\int_3^\infty \frac{3}{x+1} - \frac{3}{x+9}\,dx = \lim_{b\to\infty} 3\ln\left|\frac{x+1}{x+9}\right|_3^b =$$

$$\lim_{b\to\infty} \underbrace{3\ln\left|\underbrace{\frac{b+1}{b+9}}_{1}\right|}_{0} - 3\ln\left|\frac{4}{12}\right| = -3\ln\frac{1}{3} = 3\ln 3.$$

(e) $$L = \lim_{x\to\infty}\left(1+\frac{1}{3x}\right)^{4x} \Rightarrow \ln L = \lim \ln\left(1+\frac{1}{3x}\right)^{4x} =$$

$$\lim 4x\ln\left(1+\frac{1}{3x}\right) = \lim \frac{\ln\left(1+\frac{1}{3x}\right)}{\frac{1}{4x}} \overset{\text{LH}}{=}$$

$$\lim \frac{\frac{1}{\left(1+\frac{1}{3x}\right)} \cdot -\frac{1}{3\cancel{x^2}}}{-\frac{1}{4\cancel{x^2}}} = \lim_{x\to\infty} \frac{1}{\left(1+\frac{1}{3x}\right)} \cdot \frac{4}{3} = \frac{4}{3},$$

So $L = e^{4/3} = \sqrt[3]{e^4}$

$\boxed{2}$ (a) $\int \sin^3 x \cos^6 x \, dx = \int (1-\cos^2 x)\cos^6 x \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$= \int -(1-u^2)u^6 \, du = \int u^8 - u^6 \, du = \frac{u^9}{9} - \frac{u^7}{7} + C$

$= \frac{\cos^9 x}{9} - \frac{\cos^7 x}{7} + C$

(b) $\int \sec^4 x \tan^2 x \, dx = \int (\tan^2 x + 1)(\tan^2 x)(\sec^2 x)$

$u = \tan x \qquad du = \sec^2 x \, dx$

$= \int (u^2+1)u^2 \, du = u^5/5 + u^3/3 + C = \frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + C$

(c) $\int \underbrace{\arctan x}_{u} \underbrace{dx}_{dv} = x \arctan x - \int \frac{x}{1+x^2} dx$

$du = \frac{1}{1+x^2} \qquad v = x$

$= x \arctan x - \frac{1}{2}\ln|1+x^2| + C$

$\boxed{3}$ (a) $\int \frac{x^3}{\sqrt{x^2+9}} dx$

$x = 3\tan\theta$

$dx = 3\sec^2\theta \, d\theta$

$\sqrt{x^2+9} = 3\sec\theta$

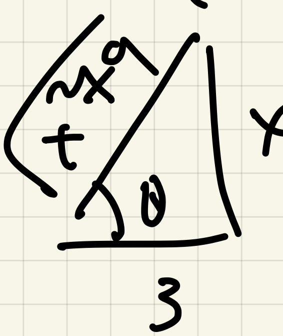
$\parallel$

$$\int \frac{(3\tan\theta)^3\ 3\sec^2\theta}{3\sec\theta} = 27\int \tan^3\theta\sec\theta\,d\theta =$$

$$27\int (\sec^2\theta - 1)\sec\theta\tan\theta = 27\int u^2-1\,du =$$

$u = \sec\theta$

$$27\left(\frac{u^3}{3} - u\right) + C = 27\left(\frac{\sec^3\theta}{3} - \sec\theta\right) + C =$$



$$27\left(\frac{\left(\frac{\sqrt{x^2+9}}{3}\right)^3}{3} - \frac{\sqrt{x^2+9}}{3}\right) =$$

$$\frac{(x^2+9)^{3/2}}{3} - 9\sqrt{x^2+9} + C$$

(b) $I = \int \underbrace{e^{-2x}}_{u}\underbrace{\cos 5x\,dx}_{dv}$ $\quad du = -2e^{-2x}$, $\ v = \frac{1}{5}\sin 5x$

$$\frac{1}{5}e^{-2x}\sin 5x + \frac{2}{5}\int \underbrace{e^{-2x}}_{u}\underbrace{\sin 5x\,dx}_{dv}$$ $\quad du = -2e^{-2x}$, $\ v = -\frac{1}{5}\cos 5x$

$$= \underbrace{\frac{1}{5}e^{-2x}\sin 5x - \frac{2}{25}e^{-2x}\cos 5x} - \frac{4}{25}\underbrace{\int e^{-2x}\cos 5x}_{I}$$

$\therefore\ \frac{29}{25}I =$, $\quad I = \frac{5e^{-2x}\sin 5x - 2e^{-2x}\cos 5x}{29} + C$

(c) $\int \frac{-20x+4}{x^2(x^2+4)}dx = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4} \Rightarrow$

$$-20x+4 = Ax(x^2+4)+B(x^2+4)+(Cx+D)x^2$$

$$= \underbrace{(A+C)}_{0}x^3+\underbrace{(B+D)}_{0}x^2+\underbrace{4A}_{-20}x+\underbrace{4B}_{4} \Rightarrow$$

$$B=1,\ A=-5,\ D=-1,\ C=5$$

$$\int -\frac{5}{x}+\frac{1}{x^2}+\frac{5x}{x^2+4}-\frac{1}{x^2+4}\,dx =$$

$$-5\ln|x|-\frac{1}{x}+5/2\ln|x^2+4|+\frac{1}{2}\arctan\frac{x}{2}+C.$$