Exam 2 (white)

$\boxed{1}$ (a) $\displaystyle\int \frac{x^2}{\sqrt{25-x^2}}\,dx$ $\qquad x = 5\sin\theta$, $\quad dx = 5\cos\theta$, $\quad \sqrt{25-x^2} = 5\cos\theta$

$$\int \frac{25\sin^2\theta \; 5\cos\theta}{5\cos\theta}\,d\theta = 25\int \sin^2\theta\,d\theta$$

(b) $\displaystyle \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+9} + \frac{Ex+F}{(x^2+9)^2} + \frac{G}{x-1} + \frac{H}{x+1}$

(c) Converges because $p = 5/3 > 1$

(d) $\displaystyle\int_4^\infty \frac{2}{x+2} - \frac{2}{x+8} = \lim_{b\to\infty} 2\ln\left|\frac{x+2}{x+8}\right|\Bigg|_4^b =$

$$\lim_{b\to\infty} 2\ln\left|\underbrace{\frac{b+2}{b+8}}_{\to 1}\right| - 2\ln\left|\frac{6}{12}\right| = 2\ln 2$$

(e) $\displaystyle L = \lim_{x\to\infty}\left(1-\frac{1}{2x}\right)^{5x} \Rightarrow \ln L = \lim_{x\to\infty} 5x\ln\left(1-\frac{1}{2x}\right)$

$$= \lim_{x\to\infty} \frac{\ln\left(1-\frac{1}{2x}\right)}{\frac{1}{5x}} \overset{0/0}{\underset{\text{LH}}{=}} \lim_{x\to\infty} \frac{\frac{1}{\left(1-\frac{1}{2x}\right)}\cdot\frac{1}{2x^2}}{-\frac{1}{5x^2}} =$$

$$\lim_{x\to\infty} \frac{1}{\left(1-\frac{1}{2x}\right)}\,\frac{-5}{2} = -5/2 \Rightarrow L = e^{-5/2}$$

$\boxed{2}$ (a) $\int \sin^4 x \cos^3 x\,dx = \int \sin^4 x(1-\sin^2 x)\cos x\,dx$

$u = \sin x$

$= \int u^4(1-u^2)\,du = \int u^4 - u^6\,du = \frac{u^5}{5} - \frac{u^7}{7} + C =$

$\frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$

$u = \sec x$

(b) $\int \tan^3 x \sec^2 x\,dx = \int (\sec^2 x - 1)\sec^2 x \sec x \tan x$

$= \int u^4 - u^2\,du = \frac{u^5}{5} - \frac{u^3}{3} + C = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$

(c) $\int \underbrace{\arcsin x}_{u}\,\underbrace{dx}_{dv} = x\arcsin x - \int \frac{x}{\sqrt{1-x^2}} =$

$du = \frac{1}{\sqrt{1-x^2}}$ $v = x$

$u = 1-x^2$
$du = -2x\,dx$

$x\arcsin x + \sqrt{1-x^2} + C$

$\boxed{3}$ (a) $\int \frac{x^3}{\sqrt{x^2-4}}\,dx$ $x = 2\sec\theta$ $dx = 2\sec\theta\tan\theta$

$\sqrt{x^2-4} = 2\tan\theta$

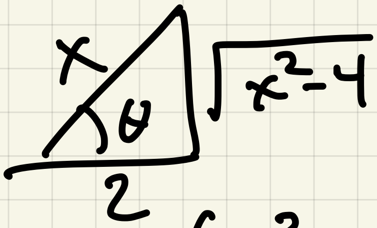
$\int \frac{8\sec^3\theta \cdot \cancel{2}\sec\theta\,\cancel{\tan\theta}}{\cancel{2}\,\cancel{\tan\theta}}\,d\theta = 8\int \sec^4\theta\,d\theta$

$u = \tan\theta$

$= 8\int (\tan^2\theta + 1)\sec^2\theta\,d\theta = 8\int u^2 + 1\,du =$

$\frac{8}{3}u^3 + 8u + C = \frac{8}{3}\tan^3\theta + 8\tan\theta + C$

$= \frac{8}{3}\left(\frac{\sqrt{x^2-4}}{2}\right)^3 + 8\left(\frac{\sqrt{x^2-4}}{2}\right) + C$



$$= \frac{(x^2-4)^{3/2}}{2} + 4\sqrt{x^2-4} + C$$

(b) $$\int \underbrace{e^{3x}}_{dv}\,\underbrace{\sin 4x}_{u}\,dx = \frac{1}{3}e^{3x}\sin 4x - \frac{4}{3}\int \underbrace{e^{3x}}_{dv}\,\underbrace{\cos 4x}_{u}\,dx =$$

$v = \frac{1}{3}e^{3x}$ $du = 4\cos 4x$

$v = \frac{1}{3}e^{3x}$ $du = -4\sin 4x$

$$\frac{1}{3}e^{3x}\sin 4x - \frac{4}{3}\left(\frac{1}{3}e^{3x}\cos 4x + \frac{4}{3}\int e^{3x}\sin 4x\right) =$$

$$\frac{1}{3}e^{3x}\sin 4x - \frac{4}{9}e^{3x}\cos 4x - \frac{16}{9}\int e^{3x}\sin 4x$$

$$\therefore \frac{25}{9}\int e^{3x}\sin 4x = \frac{1}{3}e^{3x}\sin 4x - \frac{4}{9}e^{3x}\cos 4x$$

$$\int e^{3x}\sin 4x = \frac{e^{3x}(3\sin 4x - 4\cos 4x)}{25} + C$$

(c) $$\int \frac{12x-12}{x^2(x^2+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4} \Rightarrow$$

$$12x - 12 = Ax(x^2+4) + B(x^2+4) + (Cx+D)x^2$$

$$= \underbrace{(A+C)}_{0}x^3 + \underbrace{(B+D)}_{0}x^2 + \underbrace{4A}_{12}x + \underbrace{4B}_{-12}$$

$$\Rightarrow A = 3,\ B = -3,\ C = -3,\ D = 3 \Rightarrow$$

$$\int \frac{3}{x} - \frac{3}{x^2} - \frac{3x}{x^2+4} + \frac{3}{x^2+4}\, dx =$$

$$3\ln|x| + \frac{3}{x} - \frac{3}{2}\ln|x^2+4| + \frac{3}{2}\arctan\frac{x}{2} + C$$