Exam 2

1 (a) $x = 3\sec\theta$, $dx = 3\sec\theta\tan\theta \Rightarrow \int \frac{9\sec^2\theta\, 3\sec\theta\tan\theta\, d\theta}{3\tan\theta}$

$\sqrt{x^2-9} = 3\tan\theta = \int 9\sec^3\theta\, d\theta$

(b) $\frac{A}{x+4} + \frac{B}{(x+4)^2} + \frac{Cx+D}{x^2+4} + \frac{E}{x-2} + \frac{F}{x+2}$

(c) Converges to $\frac{1}{p-1} = 2$ (d) diverges $p = 2/3 \leq 1$

(e) $L = \lim_{x\to 0^+} (1-3x)^{1/2x} \Rightarrow \ln L = \lim_{x\to 0^+} \frac{\ln(1-3x)}{2x}$

$\overset{LH}{=} \lim_{x\to 0^+} \frac{\frac{1}{(1-3x)}\cdot(-3)}{} = \frac{-3}{2} \Rightarrow L = e^{-3/2}$

(f) $\int_2^\infty \frac{3}{x} - \frac{3}{x+4} = \lim_{b\to\infty} 3\ln\left|\frac{x}{x+4}\right|_2^b =$

$\lim_{b\to\infty} 3\ln\left|\frac{b}{b+4}\right| - 3\ln\left|\frac{2}{6}\right|$

$\lim_{b\to\infty} \frac{b}{b+4} \overset{LH}{=} \lim_{b\to\infty} \frac{1}{1} = 1$, so

$\lim = 3\ln\left(\frac{1}{1}\right) - 3\ln\frac{1}{3} = \ln 3^3 = \ln 27$

2 (a) $\int \cos^4 x \sin^5 x\, dx = \int \cos^4 x (1-\cos^2 x)^2 \sin x\, dx$

$u = \cos x$, $du = -\sin x\, dx$ $= -\int u^4(1-u^2)^2\, du = \int -u^8 + 2u^6 - u^4\, du$

$= -\frac{u^9}{9} + \frac{2u^7}{7} - \frac{u^5}{5} = -\frac{\cos^9 x}{9} + \frac{2\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

(b) $\int \tan^4 x \sec^2 x\,dx = \int (\tan^4 x)(\tan^2 x + 1)\sec^2 x\,dx$

$u = \tan x$

$= \int u^4(u^2+1)\,du = \frac{u^7}{7} + \frac{u^5}{5} + C = \frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C$

(c) $\int \underbrace{\arctan(3x)}_{u}\underbrace{dx}_{dv} = x\arctan(3x) - \int \frac{3x}{1+9x^2} =$

$du = \frac{3}{1+9x^2}$ $\quad v = x$

$u = 1+9x^2$

$du = 18x\,dx$

$\frac{1}{6}du = 3x\,dx$

$x\arctan x - \frac{1}{6}\ln|1+9x^2| + C$

[3] (a) $\int \frac{x^3}{\sqrt{x^2+4}}\,dx$ $\quad x = 2\tan\theta$ $\quad dx = 2\sec^2\theta$

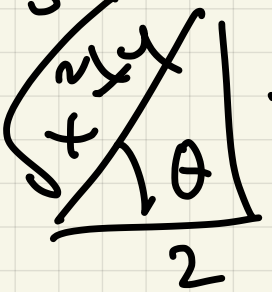
$\sqrt{x^2+4} = 2\sec\theta$

$u = \sec\theta$

$\int 8\tan^3\theta \sec\theta\,d\theta = 8\int (\sec^2\theta + 1)\sec\theta\tan\theta$

$= 8\int (u^2-1)\,du = 8/3\,u^3 - 8u + C =$

$\frac{8}{3}\sec^3\theta - 8\sec\theta + C =$



$\frac{8}{3}\left(\frac{\sqrt{x^2+4}}{2}\right)^3 - 8\left(\frac{\sqrt{x^2+4}}{2}\right) + C$

$= \frac{(x^2+4)^{3/2}}{3} - 4\sqrt{x^2+4} + C$

(b) $\int \underbrace{e^{2x}}_{dv}\underbrace{\sin 4x}_{u}\,dx = \frac{1}{2}e^{2x}\sin 4x -$

$v = \frac{1}{2}e^{2x} \quad du = 4\cos 4x$ $\int \underbrace{2e^{2x}}_{dv} \underbrace{\cos 4x}_{u} dx$

$v = e^{2x} \quad du = -4\sin 4x$

$= \frac{1}{2}e^{2x}\sin 4x - \left[e^{2x}\cos 4x + 4\int e^{2x}\sin 4x\right]$

$5\int e^{2x}\sin 4x\,dx = \frac{1}{2}e^{2x}\sin 4x - e^{2x}\cos 4x \Rightarrow$

$\int e^{2x}\sin 4x = \frac{\frac{1}{2}e^{2x}\sin 4x - e^{2x}\cos 4x}{5} + C$

(c) $\frac{18x-9}{x^2(x^2+9)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+9} \Rightarrow$

$18x - 9 = Ax(x^2+9) + B(x^2+9) + (Cx+D)x^2$

$= \underbrace{(A+C)}_{0}x^3 + \underbrace{(B+D)}_{0}x^2 + \underbrace{9A}_{18}x + \underbrace{9B}_{-9}$

$\Rightarrow A = 2,\ B = -1,\ C = -2,\ D = 1$

$\therefore \int \frac{2}{x} - \frac{1}{x^2} - \frac{2x}{x^2+9} + \frac{1}{x^2+9}\,dx =$

$2\ln|x| + \frac{1}{x} - \ln|x^2+9| + \frac{1}{3}\arctan\frac{x}{3} + C.$