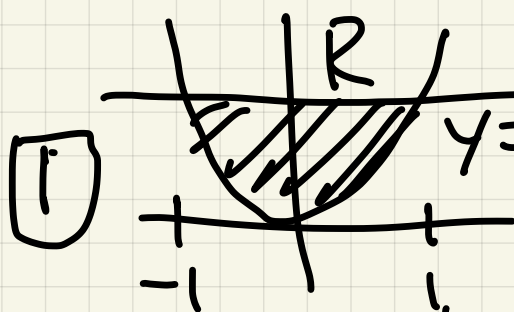


Exam 1



(1) (a) $M = \int_{-1}^1 (1 - x^2) dx = x - \frac{x^3}{3} \Big|_{-1}^1 = \frac{4}{3}$

(b) $M_y = \int_{-1}^1 x(1 - x^2) dx = \frac{1}{2}x^2 - \frac{1}{4}x^4 \Big|_{-1}^1 = 0$

$M_x = \int_{-1}^1 \frac{1^2 - (x^2)^2}{2} dx = \frac{1}{2} \left(x - \frac{x^5}{5} \right) \Big|_{-1}^1 = \frac{4}{5}$

$\therefore (\bar{x}, \bar{y}) = \left(\frac{0}{4/3}, \frac{4/5}{4/3} \right) = \left(0, \frac{3}{5} \right)$

(c) $r = 3, A = \frac{4}{3} \Rightarrow V = 2\pi r A = 2\pi \cdot 3 \cdot \frac{4}{3} = 8\pi$

[2] (a) Washers: $V = \int_0^2 \pi (e^{6x} - 1^2) dx = \pi \left(\frac{e^{6x}}{6} - x \right) \Big|_0^2 = \pi \left(\left(\frac{e^{12}}{6} - 2 \right) - \left(\frac{e^0}{6} - 0 \right) \right) = \left(\frac{e^{12} - 13}{6} \right) \pi$

(b) Shells $V = \int_0^2 2\pi x (e^{3x} - 1) dx$

(c) $V = \int_0^2 2\pi (x+3)(e^{3x} - 1) dx$

[3] (a) Shells $V = \int_1^2 2\pi y (y^3) dy = 2\pi \left(\frac{y^5}{5} \right) \Big|_1^2 = \frac{62\pi}{5}$

(b) Disks $V = \int_1^2 \pi (y^3)^2 dy$

(c) Washers $V = \int_1^2 \pi ((y^3+3)^2 - 3^2) dy$

$$[4] \quad (a) \quad y = 100 - x^2 \Rightarrow \frac{dy}{dx} = -2x \Rightarrow ds = \sqrt{1 + 4x^2}$$

$$\text{so } L = \int_0^{401} \sqrt{1 + 4x^2} \, dx$$

$$(b) \quad SA = \int_0^{401} \underbrace{2\pi x}_{\text{radius strip}} \sqrt{1 + 4x^2} \, dx = \begin{aligned} &u = 1 + 4x^2 \\ &du = 8x \, dx \\ &\frac{1}{4} du = 2x \, dx \end{aligned}$$

$$= \int_1^{401} \frac{\pi}{4} \sqrt{u} \, du = \frac{\pi}{4} \frac{2}{3} u^{3/2} \Big|_1^{401} = \frac{\pi}{6} (401^{3/2} - 1)$$

(c) cross section area at height y

$$\left. \begin{aligned} &1s \quad \pi x^2 = \pi(100 - y) \\ &\underline{\text{dist to move water}} = 100 + y \\ &\text{density} = 60 \\ &\text{Fill tank} \Rightarrow 0 \leq y \leq 100 \end{aligned} \right\} \Rightarrow$$

$$W = \int_0^{100} 60\pi(100 - y)(100 + y) \, dy =$$

$$60\pi \int_0^{100} (10000 - y^2) \, dy = 60\pi \left(10000y - \frac{y^3}{3} \right) \Big|_0^{100}$$

$$= 60\pi \left(10^6 - \frac{10^6}{3} \right) = 4000000\pi \text{ ft-lbs}$$