8/31/Calc2

Quiz 1

#1 (a) $\int 10x^4 - \frac{6}{x^4} - \frac{3}{x}\,dx$

$= 2x^5 + \frac{2}{x^3} - 3\ln|x| + C$

(b) $\int 3e^x - \frac{5}{\sqrt{9-x^2}} - 5\cos x\,dx$

$= 3e^x - 5\arcsin\frac{x}{3} - 5\sin x + C$

From formula in class, or table, or set $u = \frac{x}{3}$ substitution

$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$

#2 $\int \frac{2\cos x}{3\sin x + 1}\,dx$

$3(\sin x + 1)$ $\neq$

$u = 3\sin x + 1$

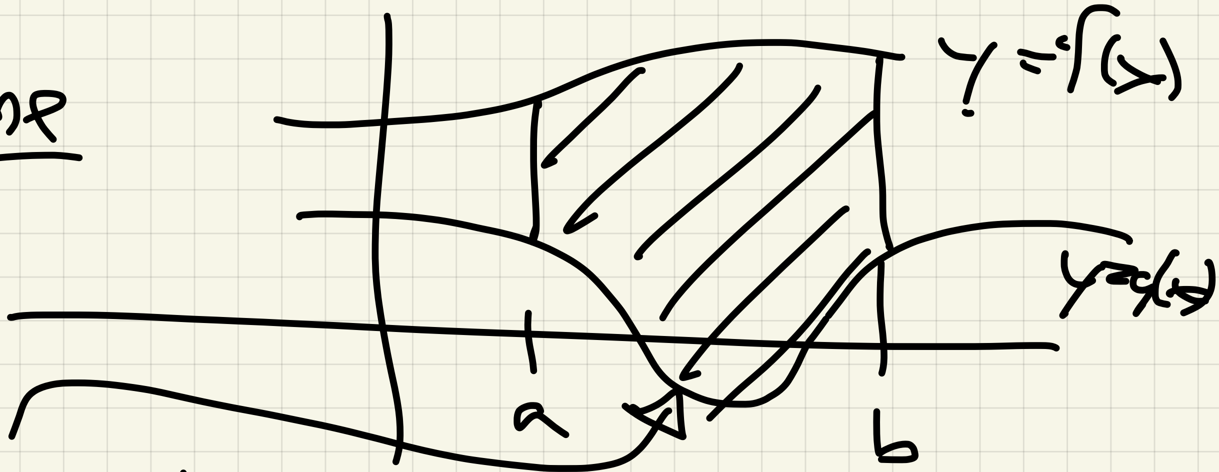
$du = 3\cos x\,dx$

$\frac{2}{3}du = 2\cos x\,dx$

$= \int \frac{\frac{2}{3}du}{u} = \frac{2}{3}\ln|u| + C$

$= \frac{2}{3}\ln|3\sin x + 1| + C$

Last time

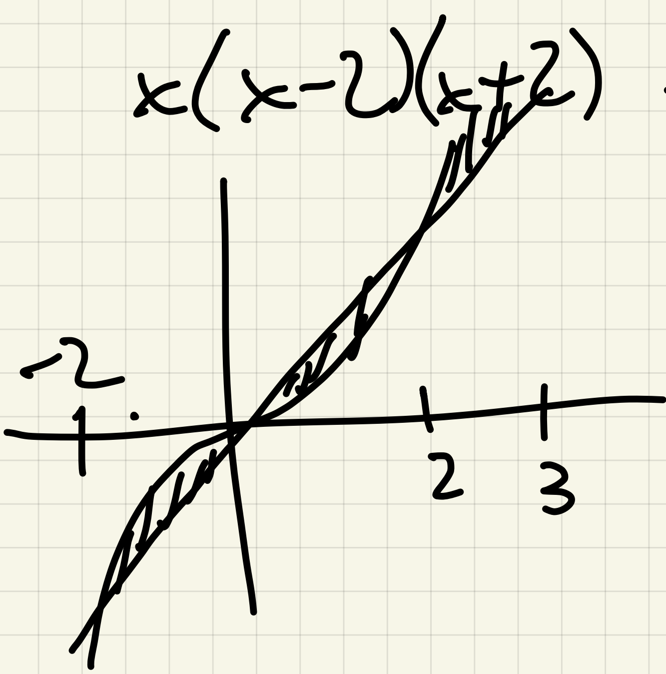


$$\text{Area} = \int_a^b f(x) - g(x)\,dx$$

Ex Set up integrals to find area bounded by $y = x^3$, $y = 4x$, $x = 3$

$x^3 = 4x \Rightarrow 0 = x^3 - 4x = x(x^2 - 4) =$

$x(x-2)(x+2) \Rightarrow x = 0, \pm 2$ $\quad \frac{x^4}{4} - 2x^2 \Big|_{-2}^{0}$



$$\text{Area} = \int_{-2}^{0} x^3 - 4x\,dx +$$

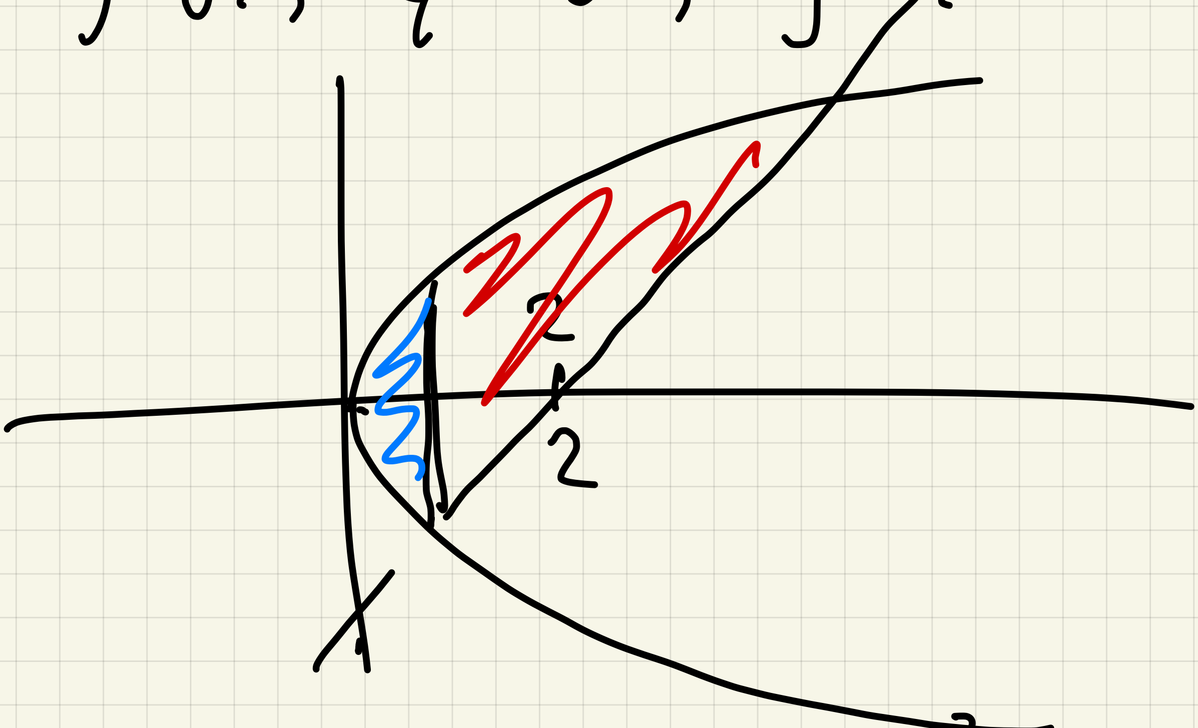
$$\int_0^2 4x - x^3\,dx +$$

$$\int_2^3 x^3-4x\,dx$$

$$= 4+4+\frac{25}{4} = \frac{57}{4} = 14\tfrac{1}{4}.$$

<u>Ex2</u> find area bounded by

$y=\sqrt{x}$, $y=-\sqrt{x}$, $y=x-2$



$$\pm\sqrt{x} = x-2 \Rightarrow x = (x-2)^2 \Rightarrow$$

$$x = x^2-4x+4 \rightarrow 0 = x^2-5x+4$$
$$= (x-1)(x-4) \rightarrow$$

$$x = 1, 4$$
$$y = -1, 2$$

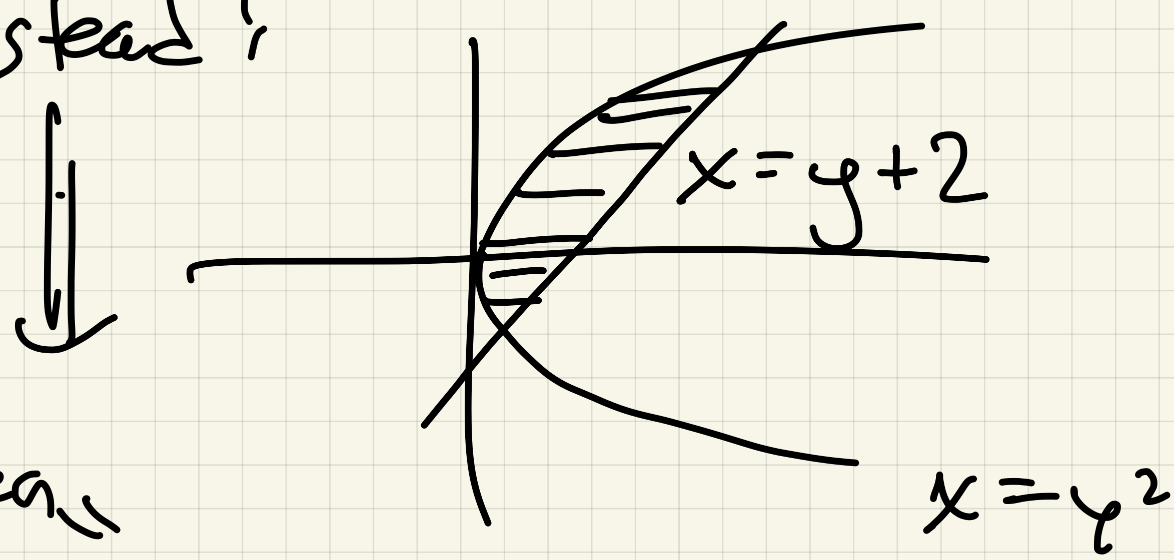
so $\text{Area} = \underbrace{\int_0^1 \sqrt{x} - (-\sqrt{x})\,dx}_{\text{blue}} + \underbrace{\int_1^4 \sqrt{x} - (x-2)}_{\text{red}}$

$= \frac{4}{3} + \frac{2}{3}x^{3/2} - \frac{x^2}{2} + 2x \Big|_1^4$

$= \frac{4}{3} + \frac{14}{3} - \frac{15}{2} + 6$

$= 9/2$

Important idea: There's a much easier way. Use y-variable instead!

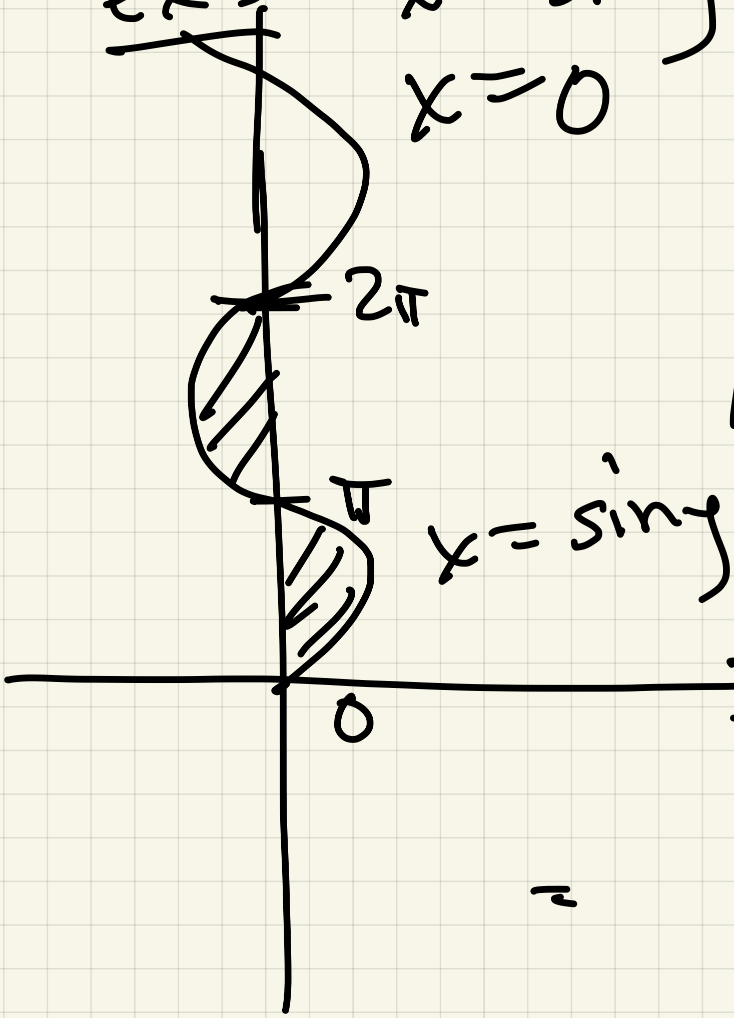


so Area =

$$\int_{-1}^{2} (y+2) - y^2 \, dy = \frac{1}{2}y^2 + 2y - \frac{y^3}{3} \Big|_{-1}^{2} =$$

$$\frac{3}{2} + 6 - \frac{9}{3} = \frac{9}{2} \checkmark$$

Ex 3 $\quad x = \sin y \quad x = 0 \quad y = 0 \quad y = 2\pi$ bounds region



$$\text{Area}_{\text{bottom}} = \int_0^{\pi} (\sin y - 0)\, dy$$

$+$

$$\cancel{\text{top}} \int_{\pi}^{2\pi} (0 - \sin y)\, dy$$

$= \quad 2 + 2 = 4.$