4/6 Calc2

last time

Ex 0 $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ — n^{th} terms test fails — Int test — telescoping

Easier way: $\frac{1}{n(n+1)} > 0 \Rightarrow$

S_n = partial sum

S_n increasing (monotone)

Need to check if S_n is bounded:

but $\frac{1}{n(n+1)} \leq \frac{1}{n^2}$

$\sum \frac{1}{n^2}$ converges: $p = 2 > 1$

$\therefore$ If t_n = partial sum for $\sum \frac{1}{n^2}$

t_n bdd, but $s_n \le t_n$, so
s_n also converges

<u>Direct Comparison test</u> (DCT)

If $0 \le a_n \le b_n$

① $\sum b_n$ converges $\to \sum a_n$ converges

② $\sum a_n$ diverges $\to \sum b_n$ diverges

<u>Ex)</u> $\displaystyle\sum_{n=2}^{\infty} \frac{3^n - 2^n}{5^n}$ (using 9.2, ~~convergent~~)

$\boxed{\dfrac{3^n - 2^n}{5^n} < \dfrac{3^n}{5^n},}$

comparison

$\sum \frac{3^n}{5^n}$ geometric, $r = 3/5 < 1$

$\therefore$ converges

$\sum \frac{3^n - 2^n}{5^n}$ conv.

Ex2 (a) $\sum_{n=1}^{\infty} \frac{5n-2}{n^3}$ (lnt test works)

but $\frac{5n-2}{n^3} < \frac{5n}{n^3} = \frac{5}{n^2}$

$5\sum_{n=1}^{\infty} \frac{1}{n^2}$ conv ($p = 2 > 1$ ✓) p-series

$\frac{5n-2}{n^3} < \frac{5}{n^2}$ ✓

$\Rightarrow \sum_{n=1}^{\infty} \frac{5n-2}{n^3}$ converges.

(b) $\sum_{n=1}^{\infty} \frac{5n+2}{n^2}$

$\frac{5n+2}{n^2} > \frac{5n}{n^2} = \frac{5}{n}$

$5\sum \frac{1}{n}$ diverges ($p = 1 \leq 1$ ✓)

DCT $\Rightarrow \sum \frac{5n+2}{n^2}$ diverges

(c) $\sum_{n=1}^{\infty} \frac{5n+2}{n^3}$ (~~Expect~~ converges)

$\frac{5n+2}{n^3} > \frac{5n}{n^3}$ inequality wrong direction

DCT not useful.

Need a bigger series converges

$\frac{5n+2}{n^3} \leq \frac{7n}{n^3}$ ✓

$7 \sum \frac{n}{n^3} = 7 \sum \frac{1}{n^2}$ conv $p = 2 > 1$

$\therefore$ DCT $\Rightarrow \sum \frac{5n+2}{n^3}$ converges

(d) $\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}}$ ~~Expect~~ diverges

$\ln n \geq 1$ if $n \geq e$

$$\boxed{\frac{\ln n}{\sqrt{n}} \geq \frac{1}{\sqrt{n}}} \quad n \geq e$$

$n \geq 3$

So $\sum_{n=3}^{\infty} \frac{1}{\sqrt{n}}$ diverg. $p = 1/2 \leq 1$

so DCT $\Rightarrow$ $\sum \frac{\ln n}{\sqrt{n}}$ diverges.

(e) $$\sum_{n=1}^{\infty} \frac{12}{\sqrt[8]{n^9+17}} \leq \frac{12}{\sqrt[8]{n^9}} =$$

$$\sum \frac{12}{n^{9/8}}$$ conv. because

$p = 9/8 > 1$

p-series

$$\therefore \sum_{n=1}^{\infty} \frac{12}{\sqrt[8]{n^9+17}}$$

(f) $\displaystyle\sum_{n=2}^{\infty} \frac{12}{\sqrt[8]{n^9 - 17}}$ Could find a tricky direct comparison...

Limit Comparison Test (LCT)

$0 < a_n, b_n$ all $n \geq N$

① $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$, then

$\sum a_n$ & $\sum b_n$ both converge OR both divergence

② $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, $\sum b_n$ conv

$\Downarrow$

$\sum a_n$ converges

③ $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = +\infty$, $\sum b_n$ diverges

$\Downarrow$

$\sum a_n$ diverges

Ex 3 (a) $\sum_{n=1}^{\infty} \frac{5n+2}{n^3} \sim \frac{n}{n^3} = \frac{1}{n^2}$

Apply LCT

$$\lim_{n\to\infty} \frac{\frac{5n+2}{n^3}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{(5n+2)(n^2)}{n^3} =$$

$$\lim_{n\to\infty} \frac{5n+2}{n} = \lim_{n\to\infty} \frac{5+\overset{\to 0}{2/n}}{1} = \frac{5}{1} \neq 0$$

$\sum \frac{1}{n^2}$ convergent ($p = 2 > 1$ p-series)

LCT $\Rightarrow$ $\sum \frac{5n+2}{n^3}$ converges

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+10}} \sim \sum \frac{1}{\sqrt{n}}$ diverges $p = \frac{1}{2} \leq 1$ p-series

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n+10}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+10}} =$$

$$\lim_{n\to\infty} \sqrt{\frac{n}{n+10}} = \sqrt{\lim \frac{n}{n+10}} = \sqrt{1} = 1 \neq 0$$

$\therefore$ LCT $\Rightarrow$ $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+10}}$ diverges.

(c) $\sum \frac{12}{\sqrt[8]{n^9-17}} \sim \sum \frac{1}{\sqrt[8]{n^9}} =$

$\sum \frac{1}{n^{9/8}}$ conv

$$\lim \frac{\frac{12}{\sqrt[8]{n^9-17}}}{\frac{1}{\sqrt[8]{n^9}}} =$$

$$\lim 12 \sqrt[8]{\frac{n^9}{n^9-17}}$$