4/23 | Calc 2

Exam 3 → Tuesday

Drop lowest 3 quiz scores

Last time

Estimates using Taylor remainder

$$f(b) = P_n(b) + \underbrace{R_n(b)}_{\text{error}}$$

Taylor poly centered at $x=c$

$$R_n(b) = \frac{f^{(n+1)}(z)}{(n+1)!}(b-c)^{n+1}$$

z between c, b

Quiz 19

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{n^2 \cdot 9^n}(x-4)^n$$

I.O.C.:

Ratio test $\lim_{n\to\infty} \left| \frac{\frac{1}{(n+1)^2 9^{n+1}} \boxed{(x-4)^{n+1}}}{\frac{1}{n^2 9^n} \boxed{(x-4)^n}} \right|$

$$= \lim_{n\to\infty} \frac{|x-4|}{9}\left(\frac{n}{n+1}\right)^2 = \frac{|x-4|}{9}$$

<u>conv</u>: $\frac{|x-4|}{9} < 1$

$$-1 < \frac{x-4}{9} < 1$$

$$-9 < x-4 < 9$$

$$-5 < x < 13$$

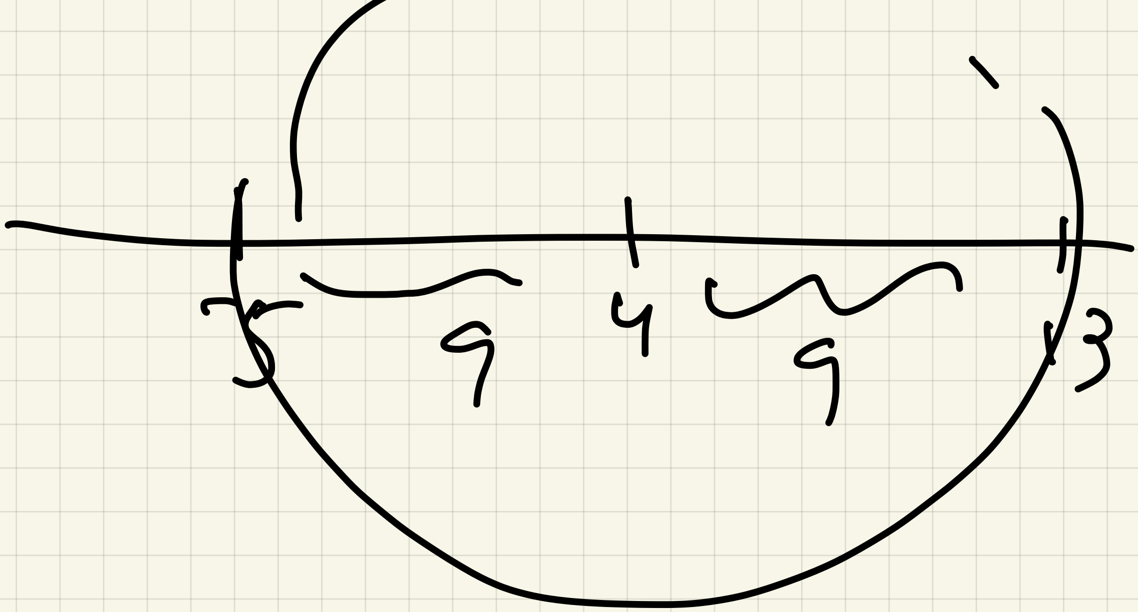
<u>Endpoint check</u>:

$x = -5 \Rightarrow \sum \frac{(-1)^n}{n^2}$ converges by AST

$x = 13 \Rightarrow \sum \frac{1}{n^2}$ converges by p-series, $p = 2 > 1$

$\underline{[-5, 13]}$

(b) $\frac{13-(-5)}{2} = \frac{18}{2} = 9$



(c)

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{n^2 9^n} \boxed{(x-4)^n}$$

$\frac{d}{dx} f$

$$f'(x) = \sum_{n=1}^{\infty} \frac{1}{n^2 \cdot 9^n} n (x-4)^{n-1}$$

$$= \sum_{n=1}^{\infty} \frac{(x-4)^{n-1}}{n \cdot 9^n}$$

$\int_0$

§9.10: Geometric series

$-1<x<1$

① $$\frac{1}{1-x} = 1+x+x^2+\dots+ = \sum_{k=0}^{\infty} x^k$$

② $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$

③ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ $\qquad$ $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

④ $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

<u>Examples:</u> ② - ④

<u>Ex①</u> Find Maclaurin Series for $f(x) = x^3 e^{-x}$ $\qquad f^{(100)}(0) = ??$

$x \leftarrow -x$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$$

$$x^3 e^{-x} = x^3 - x^4 + \frac{x^5}{2!} - \frac{x^6}{3!} + \cdots$$

$=$

$$e^x = \sum \frac{x^k}{k!} \Rightarrow e^{-x} = \sum \frac{(-x)^k}{k!} =$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k!} \Rightarrow$$

$$x^3 e^{-x} = \sum_{k=0}^{\infty} \frac{(-1)^k x^{k+3}}{k!}$$

$$x^3 e^{-x} = \underline{\underline{x^3}} - x^4 + \frac{x^5}{2!} - \frac{x^6}{3!} + -$$

$$\parallel \qquad\qquad \parallel$$

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k = f(0) + f'(0)x + \frac{f''(0)}{2!} x^2 + \ldots$$

$$\Downarrow$$

$$f(0) = 0, \quad f'(0) = 0 = f''(0)$$

$$1 = \frac{f'''(0)}{3!}$$

$$-1 = \frac{f''''(0)}{4!}$$

$$\vdots$$

$$\frac{f^{(100)}(0)}{100!} = \frac{-1}{97!} \Rightarrow$$

$$f^{(100)}(0) = -\frac{100!}{97!} =$$

$$-100 \cdot 99 \cdot 98 =$$

$$-970,200$$

Ex $f(x) = e^{x^3}$ $\quad f^{(100)}(0) = ?$

$f^{(99)}(0) = ?$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$e^{x^3} = 1 + x^3 + \frac{x^6}{2!} + \frac{x^9}{3!} + \frac{x^{12}}{4!} + \ldots$$

$$\ldots + \frac{x^{99}}{33!} + \ldots 0\,\frac{x^{100}}{1}$$

$$f^{(100)}(0) = 0$$

$$\frac{1}{33!} = \frac{f^{(99)}(0)}{99!} \leftarrow$$

$$f^{(99)}(0) = \frac{99!}{33!}$$

<u>Ex2</u> Find Maclaurin series for

(a) $f(x) = \underline{\cos(x^2)}$

(b) $\int f(x)\,dx = \ ?$

(c) $\int_0^2 f(x)\,dx$

(d) $\lim_{x \to 0} \frac{\cos(x^2) - 1}{3x^4}$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\boxed{\cos(x^2) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots}$$

$$\int \cos(x^2)\,dx = C + \underbrace{x - \frac{x^5}{5 \cdot 2!} + \frac{x^9}{9 \cdot 4!} - \frac{x^{13}}{13 \cdot 6!} + \cdots}_{g}$$

$\int_0^2 f(x)\,dx = g(2) - g(0) =$

$2 - \frac{2^5}{5 \cdot 2!} + \frac{2^9}{9 \cdot 4!} - \frac{2^{13}}{13 \cdot 6!} + \cdots$

$\lim_{x \to 0} \frac{\boxed{\cos(x^2)} - 1}{3x^4}$

$\lim_{x \to 0} \frac{\left(\cancel{1} - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots\right) - \cancel{1}}{3x^4}$

$\lim_{x \to 0} \frac{-\frac{1}{2!} + \frac{x^4}{4!} + \frac{x^8}{6!} - \cdots}{3}$ $\to 0$

$= \frac{-\frac{1}{2!}}{3} = -\frac{1}{6}$

Ex 2

~~f(x)~~ Find Maclaurin series

for $f(x) = \cos^2 x$

- COVID definition —

• COULD multiply power series

<u>Use</u> $\cos^2 x = \frac{1+\cos 2x}{2}$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$\cos(2x) = 1 - \frac{2^2x^2}{2!} + \frac{2^4x^4}{4!} - \frac{2^6x^6}{6!} + \dots$$

So

$$\frac{\cos(2x)+1}{2} = 1 - \frac{2\cdot x^2}{2!} + \frac{2^3x^4}{4!} - \frac{2^5x^6}{6!} + \dots$$

<u>Ex3</u> $\sinh x = \frac{e^x - e^{-x}}{2}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

<u>subtract</u> $= 2x + \frac{2x^3}{3!} + \frac{2x^5}{5!} + \dots$

$$\frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

Ex 4

$$\boxed{i = \sqrt{-1},\quad i^2 = -1,\quad i^3 = -i,\quad i^4 = 1,\quad i^5 = i}$$

First series for $e^{ix} =$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{ix} = 1 + ix + \frac{-x^2}{2!} - \frac{x^3}{3!}i + \frac{x^4}{4!} + i\frac{x^5}{5!}$$

$\cos x + i \sin x$