4/23/Calc2 Quiz

(a) $\sum_{n=1}^{\infty} \frac{n^2}{(-5)^n}$ Ratio $\lim_{n\to\infty} \left| \frac{\frac{(n+1)^2}{(-5)^{n+1}}}{\frac{n^2}{(-5)^n}} \right|$

$\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^2 \frac{1}{5} = \frac{1}{5} < 1$ Ratio

$\frac{n+1}{n} \to 1$

$\therefore$ Conv absolutely

(b) $\sum_{n=2}^{\infty} \left(\frac{4n}{5n-3}\right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{4n}{5n-3}\right)^n\right|} =$

$\lim_{n\to\infty} \frac{4n}{5n-3} = \frac{4}{5} < 1$

$\therefore$ converges by root test.

(c) $\sum_{n=1}^{\infty} \frac{n!}{5^n}$

$$\lim_{n\to\infty}\left|\frac{\frac{(n+1)!}{5^{n+1}}}{\frac{n!}{5^n}}\right| = \lim_{n\to\infty}\frac{(n+1)!}{n!}\cdot\frac{5^n}{5^{n+1}}$$

$$\frac{5^n}{5^{n+1}} = \frac{1}{5}$$

$$= \lim_{n\to\infty}\frac{n+1}{5} = +\infty > 1$$

diverges by ratio test.

Last time

$f(x)$ = function

c = center

Taylor polynomial

$$P_n(x) = \sum_{k=0}^{n}\frac{f^{(k)}(c)}{k!}(x-c)^k$$

Taylor series

$$\sum_{k=0}^{\infty}\frac{f^{(k)}(c)}{k!}(x-c)^k$$

Taylor theorem

If $f^{(n+1)}$ exists on open interval containing c and b, then there is a number z between c and b with

$$f(b) = \underbrace{P_n(b)}_{\text{Taylor}} + \boxed{\underbrace{\frac{f^{(n+1)}(z)}{(n+1)!}(b-c)^{n+1}}_{R_n(b) = \text{remainder}}}$$

error

Ex 1 For $c = 0$, $\underline{n = 5}$, $f(x) = \sin x$

Estimate $f(1) = \sin 1$

(a) using $P_5(1)$

(b) Use Taylor's Theorem to estimate error

(c) Find actual error

(a)

k	$f^{(k)}(x)$	$f^{(k)}(0)/k!$
0	$\sin x$	0
1	$\cos x$	$1/1!$
2	$-\sin x$	0
3	$-\cos x$	$-1/3!$
4	$\sin x$	0
5	$\cos x$	$1/5!$

$\Rightarrow$

$$P_5(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$$

(b) $\sin 1 = f(1) \approx P_5(1) =$

$b = \cancel{0}$

$c = 0$

$$1 - \frac{1}{3!} + \frac{1}{5!} = \frac{101}{120} \approx .841\overline{6}$$

Worst error:

$$\frac{f^{(6)}(z)}{6!}(1-0)^6$$

$$\left|\frac{f^{(6)}(z)}{6!}(1-0)^6\right| \leq \frac{1}{6!} = .0013\overline{8}$$

1

Actual error:

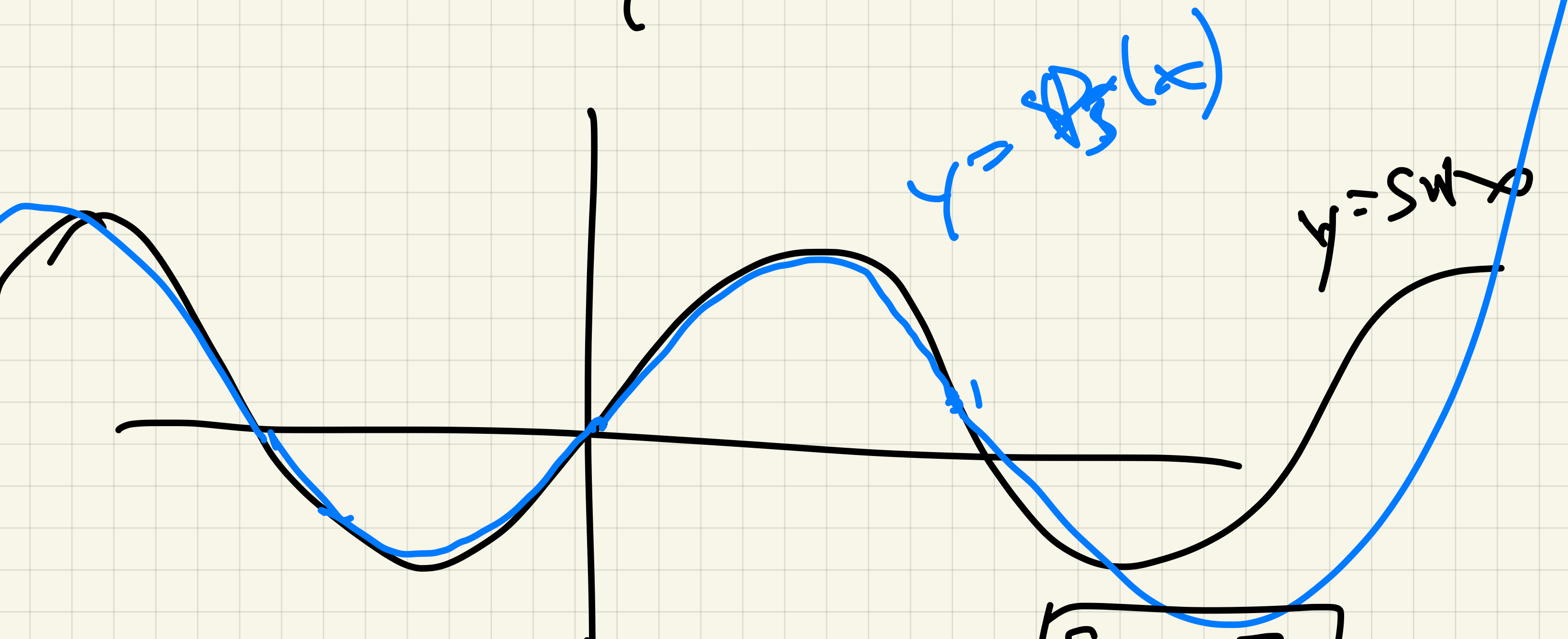
$$\left|\sin 1 - \frac{101}{120}\right| = 1.95\times 10^{-4}$$

$$= .000195 < .0013\overline{8}$$

Calculator

$y = \sin x$

$y = P_5(x)$



Ex2 Bound error on $\boxed{[-2,2]}$ of $\sin x$ with $P_9(x)$

$$P_n(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$$

$$|R_9(x)| \le \left| \frac{(\sin z)}{10!} (x-0)^{10} \right| \le$$

$$\to -2 \le x \le 2$$

$$\frac{2^{10}}{10!} = 2.82 \times 10^{-4}$$

Ex) Estimate $\sqrt{101}$ with $P_2(x)$

$f(x) = \sqrt{x}$, $c = 100$

(a) Find $P_2(x)$

k	$f^{(k)}(x)$	$f^{(k)}(100)/k!$	
0	$\sqrt{x}$	$\sqrt{100} = 10/1$	
1	$\frac{1}{2} x^{-1/2}$	$\frac{1}{20}$	
2	$-\frac{1}{4} x^{-3/2}$	$-\frac{1}{4000 \cdot 2!}$	$-\frac{1}{9 \cdot 10^3}$
3			

$$-\frac{1}{4} 100^{-3/2} = -\frac{1}{4 \cdot 100^{3/2}}$$

$$P_2(x) =$$

$$10 + \frac{1}{20}(x-100) - \frac{1}{8000}(x-100)^2$$