4/13 Calc2 : Quiz 15

(a) $\sum \frac{k+2}{8k+5000}$

$$\lim_{n\to\infty} \frac{n+2}{8n+5000} = \lim_{n\to\infty} \frac{1+\left(\overset{\to 0}{2/n}\right)}{8+\left(\underset{\to 0}{5000/n}\right)} = \frac{1}{8} \neq 0$$

$\therefore$ series diverges

(b) $\sum_{k=2}^{\infty} 18\left(\frac{2}{3}\right)^k$

$$18\left(\frac{2}{3}\right)^2 + 18\left(\frac{2}{3}\right)^3 + 18\left(\frac{2}{3}\right)^4 + \ldots$$

Geom, $r = \frac{2}{3} < 1 \Rightarrow$ convergent

converges to $\frac{a}{1-r} = \frac{18\left(2/3\right)^2}{1-2/3} =$

$\frac{8}{1/3} = 24$

$\boxed{2}$ $\displaystyle\sum_{k=1}^{\infty} \frac{9}{(3k-1)(3k+2)} = \underset{k=1}{\frac{9}{10}} + \underset{k=2}{\frac{9}{40}} + \underset{k=3}{\frac{9}{88}} + \dots$

(a)

$S_1 = \frac{9}{10}$ $\quad S_2 = \frac{9}{10} + \frac{9}{40} = \frac{45}{40} = \frac{9}{8}$

$S_3 = \frac{9}{10} + \frac{9}{40} + \frac{9}{88} = \frac{27}{22}$

(b) $\displaystyle \frac{9}{(3k-1)(3k+2)} = \underbrace{\frac{3}{3k-1} + \frac{3}{3k+2}}_{} = a_k$

$S_n = a_1 + a_2 + a_3 \dots + a_n$

$= \underset{a_1}{\left(\frac{3}{2} - \cancel{\frac{3}{5}}\right)} + \underset{a_2}{\left(\cancel{\frac{3}{5}} - \cancel{\frac{3}{8}}\right)} + \left(\cancel{\frac{3}{8}} - \cancel{\frac{3}{11}}\right) + \dots$

$\left(\cancel{\frac{3}{3n-1}} - \frac{3}{3n+2}\right)$

$S_n = \frac{3}{2} - \frac{3}{3n+2} \to 0$

$\text{Sum} = \lim_{n\to\infty} S_n = 3/2$

Quiz 16

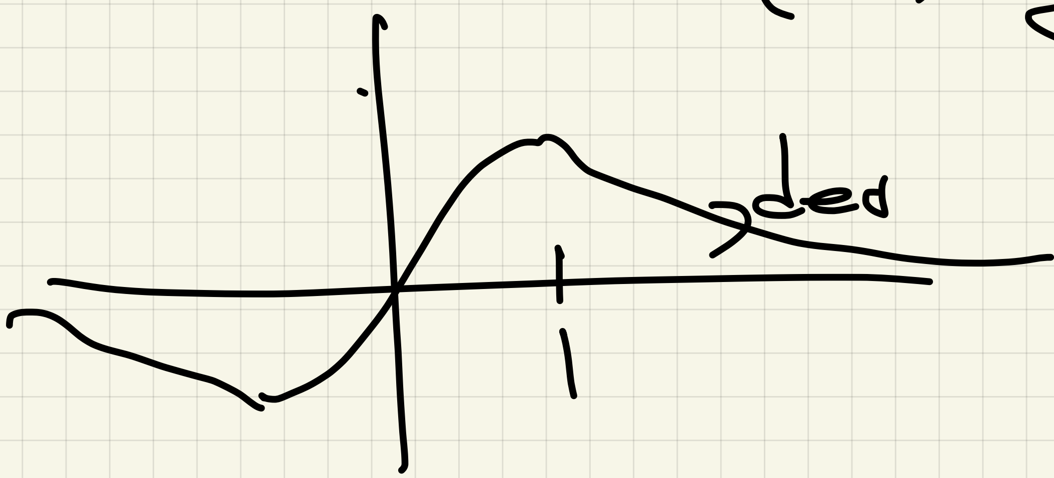
$$\sum_{k=1}^{\infty} \frac{4k}{k^2+1} \sim \quad f(x) = \frac{4x}{x^2+1}$$

cont ✓ pos ✓

decr ✓ $f'(x) = \dfrac{4(x^2+1) - 2x(4x)}{(x^2+1)^2}$

$$= \frac{4x^2+4-8x^2}{(x^2+1)^2}$$

$$\frac{4-4x^2}{(x^2+1)^2} \leq 0 \quad x \geq 1$$



$$\int_1^{\infty} \frac{4x}{x^2+1}\,dx = \lim_{b\to\infty} 2\ln(x^2+1)\Big|_1^b$$

$$= \lim_{b\to\infty} 2\ln(b^2+1) - 2\ln 2$$

$$= +\infty$$

Calculus Bee: 5-6
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Series tests: n^{th} term test

$\boxed{2}$ $\sum_{k=1}^{\infty} \frac{1}{k^4}$ p-series $p = 4 > 1$ $\therefore$ converges

$\boxed{3}$ $\sum \frac{3k^2}{8k^2+1}$ $\lim_{n\to\infty} \frac{3n^2}{8n^2+1} \overset{L'H}{=} \frac{3}{8} \neq 0$

∴ diverges
n^{th} term

$\boxed{4}$ $$\sum_{k=1}^{\infty} \frac{5}{3^k} = \frac{5}{3} + \frac{5}{3^2} + \frac{5}{3^3}$$

$r = \frac{1}{3} < 1$
geom
$\rightarrow$ converges

Series tests:

- n^{th} term test
- geom
- integral
- DCT
- LCT
- Abs conv test
- Ratio
- Root
- AST

Last time: Power series:

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n =$$

$$a_0 + a_1(x-c) + a_2(x-c)^2 + \cdots$$

c = center, a_n = coefficients

I.O.C. = Interval of convergence

$= \{x : f(x) \text{ convergent}\}$

Ex 0 $$f(x) = \sum_{n=0}^{\infty} \frac{1}{5^n}(x-4)^n =$$

coeffs center

$$1 + \frac{1}{5}(x-4) + \frac{1}{25}(x-4)^2 + \cdots$$

Geometric, $r = \frac{1}{5}(x-4)$

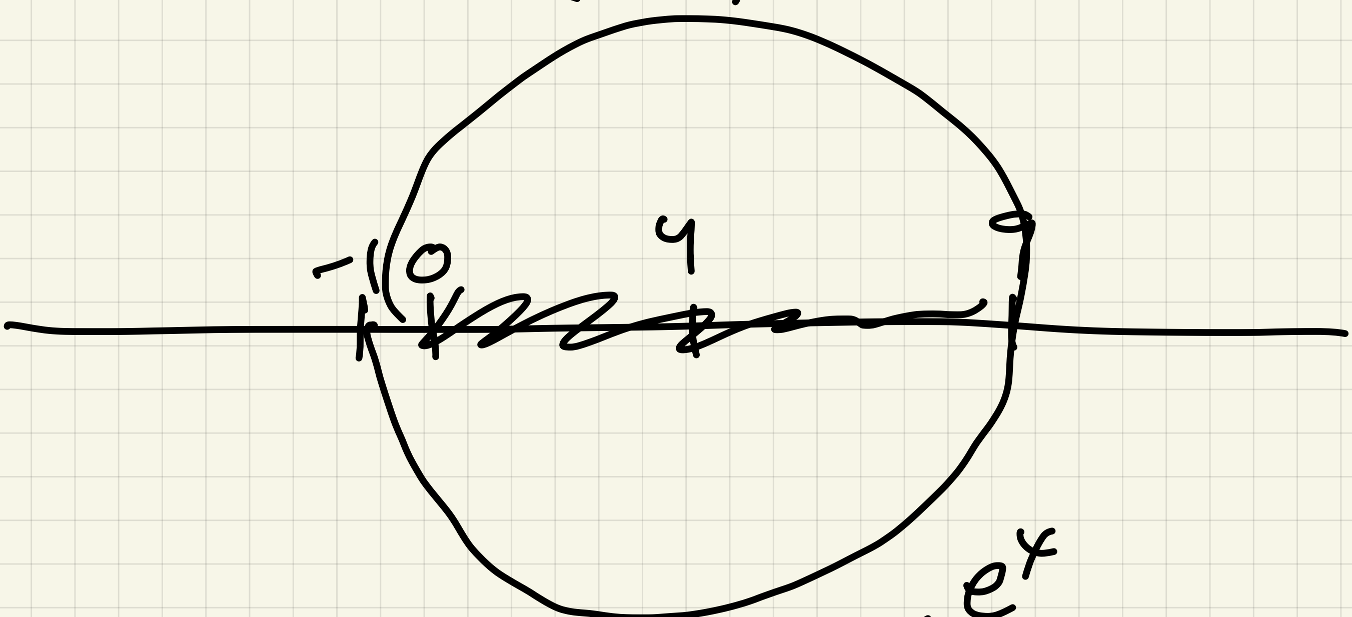
converges for

$$-1 < \frac{1}{5}(x-4) < 1$$

$$-5 < x-4 < 5$$

$$\underbrace{-1 < x < 9}$$

I.O.C. $(-1, 9)$



Ex1 (a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!} x^n = e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots$

$$\lim_{x \to \infty} \left| \frac{\frac{1}{(n+1)!} \, x^{n+1}}{\frac{1}{n!} \, x^n} \right| =$$

$$= \lim_{x \to \infty} \left| \frac{n!}{(n+1)!} \right| \left| \frac{x^{n+1}}{x^n} \right|$$

$$= \lim_{n \to \infty} \frac{1}{n+1} |x| = 0 < 1$$

all x

$\therefore$ f. o.c,

$(-\infty, \infty)$

(h) $$\sum_{n=0}^{\infty} \frac{(x+1)^n}{3^n \sqrt{n+10}}$$ $\boxed{c = -1}$

Ratio:

$$\lim_{n \to \infty} \left| \frac{\dfrac{(x+1)^{n+1}}{3^{n+1}\sqrt{n+11}}}{\dfrac{(x+1)^n}{3^n \sqrt{n+10}}} \right| =$$

$$\lim_{n\to\infty}\left|\frac{(x+1)^{n+1}}{(x+1)^n}\right|\left|\frac{3^n}{3^{n+1}}\right|\left|\frac{\sqrt{n+10}}{\sqrt{n+11}}\right|.$$

$$= \lim_{n\to\infty}|x+1|\,\frac{1}{3}\sqrt{\frac{n+10}{n+11}} =$$

$$\frac{n+10}{n+11} \to 1$$

$$r = \frac{|x+1|}{3}$$

converges $\quad -1 < \frac{x+1}{3} < 1$

$$-3 < x+1 < 3$$

$$-4 < x < 2$$

converges $\quad (-4, 2)$

diverges $\quad x < -4, \; x > 2$

Endpoints: check end points:

$x = -4, \; x = 2$