Artin groups via mapping class groups

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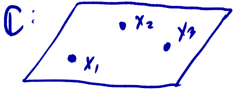
Braid groups and Hyperplane arrangements

Let $V = \mathbb{C}^3$ with coordinates (x_1, x_2, x_3). The type A_2 reflection arrangement is

$$H_{12} = \{x_1 = x_2\}, \quad H_{13} = \{x_1 = x_3\}, \quad H_{23} = \{x_2 = x_3\}.$$

The complement is $M = V \setminus (H_{12} \cup H_{13} \cup H_{23})$.

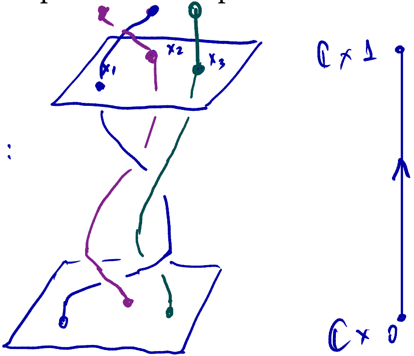
M is the configuration space of *ordered* triples of *distinct* points in $\mathbb{C}$.



A pure braid is a loop in M:

$f\ [0,1] \longrightarrow M$

A braid is a loop in M/S_3



$P_{A_2} = \pi_1(M)$ (pure braids); $\quad A_{A_2} = \pi_1(M/S_3) \cong B_3$(all braids on 3 strands)

From arrangements to Artin groups

- **Reflection setup:** finite Coxeter group W on $V \cong \mathbb{C}^n$; reflecting hyperplanes $\{H_s\}$.
- **Pure vs. full:**

$$P_W = \pi_1\big(V \setminus \bigcup H_s\big), \qquad A_W = \pi_1\big((V \setminus \bigcup H_s)/W\big),$$

fitting in the exact sequence $\quad 1 \to P_W \to A_W \to W \to 1.$

Here A_W **Artin group** with Coxeter group W

P_W **pure Artin group**

Classical reflection arrangements: defining hyperplanes

Let $V = \mathbb{C}^n$ with coordinates $(x_1, \ldots, x_n)$.

- **Type** A_{n-1} (braid): $x_i - x_j = 0 \quad (1 \leqslant i < j \leqslant n)$.
- **Type** B_n (hyperoctahedral): $x_i = 0$ $(1 \leqslant i \leqslant n)$ and $x_i \pm x_j = 0$ $(1 \leqslant i < j \leqslant n)$.
- **Type** D_n: $x_i \pm x_j = 0$ $(1 \leqslant i < j \leqslant n)$ (no coordinate hyperplanes $x_i = 0$).
- **Dihedral** $I_2(m)$: m lines through the origin in $\mathbb{C}^2$:

Pure vs. full Artin groups. For any finite W with reflecting hyperplanes $\{H_s\}$,

$$P_W = \pi_1\big(V \setminus \bigcup H_s\big), \qquad A_W = \pi_1\big((V \setminus \bigcup H_s)/W\big),$$

Also **exceptional arrangements**: $E_6, E_7, E_8, F_4, H_3, H_4$, defined by explicit hyperplane constructions.

Artin groups: general definition

Coxeter gp $= A/\langle s^2 \mid s \in S \rangle$

- S finite set of generators
- $M = (m_{s,t})_{s,t \in S}$ a symmetric matrix of $\{0, 1, \ldots, \infty\}$

The **Artin group** corresponding to (S, M) is given by the presentation:

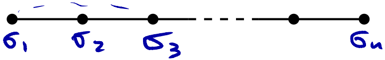
$$A = \Big\langle\ S\ \Big|\ \underbrace{stst\ldots}_{m_{s,t}} = \underbrace{tsts\ldots}_{m_{s,t}}\ ,\quad \forall s,t \in S,\ s,t \neq \infty\ \Big\rangle$$

encoded by the **Coxeter graph** with vertex set S and edges labeled by $m_{s,t}$:

Convention: $m_{s,t} = 2$: no edge, $m_{s,t} = 3$: label omitted

$st = ts$ $\qquad$ $sts = tst$

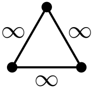
Examples:



braid group (A_n)

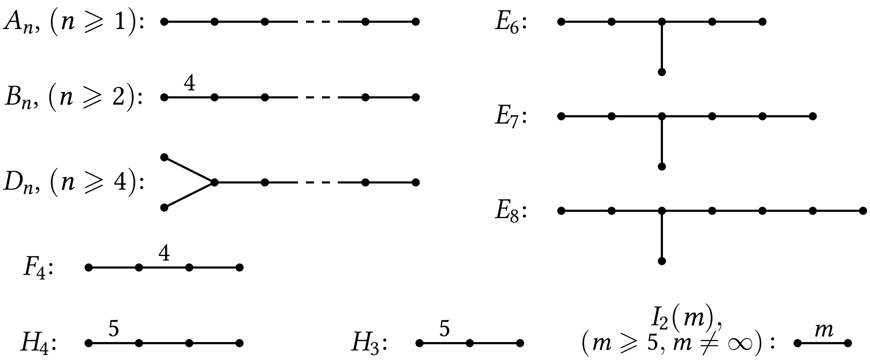
$\sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2$

$\sigma_1\sigma_3 = \sigma_3\sigma_1$

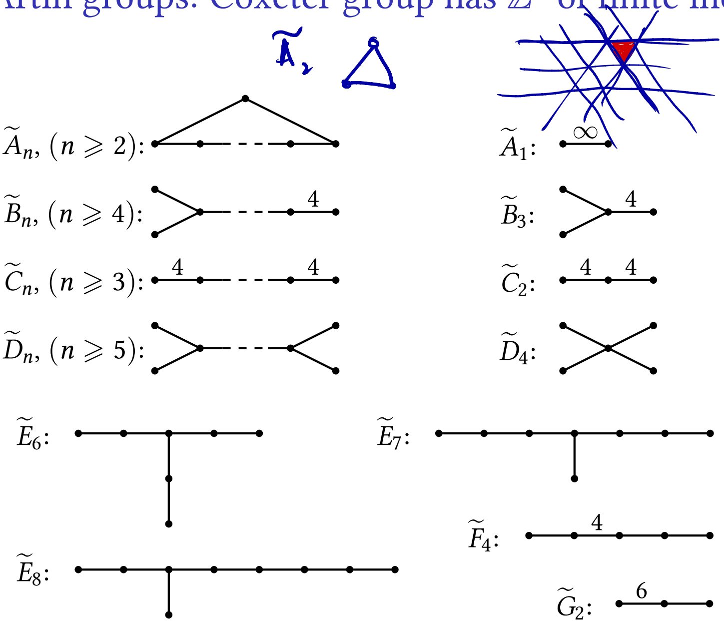


free group F_3

Spherical Artin groups



Affine Artin groups: Coxeter group has $\mathbb{Z}^n$ of finite index



Classical problems: Spherical vs Affine

- word problem, torsion-free, center: Brieskorn–Saito'1972, McCammond–Sulway'2017
- $K(\pi, 1)$-conjecture holds: Deligne'1972, Paolini–Salvetti'21

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Problem: Description of $\mathrm{Aut}(G)$, $\mathrm{Out}(G)$:

– $\widetilde{A}_1 = F_2$: Nielsen'1918: $\mathrm{Out}(A(\widetilde{A}_1)) = GL(2, \mathbb{Z})$;

– A_n: Dyer–Grossman'1981;

– $I_2(m)$: Gilbert–Howie–Metaftsis–Raptis'2000, Crisp–Paris'2002;

– A_n, B_n, $\widetilde{A}_n$, $\widetilde{C}_n$: Charney–Crisp'2005;

– D_n ($n = 4$, $n \geqslant 6$): S.'21, Castel–Paris'2025+

That is all we know!

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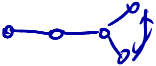

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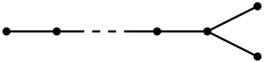
A_n:

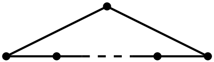
B_n:

$\widetilde{C}_n$:

D_n:



$\widetilde{A}_n$:



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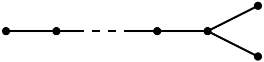

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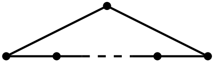

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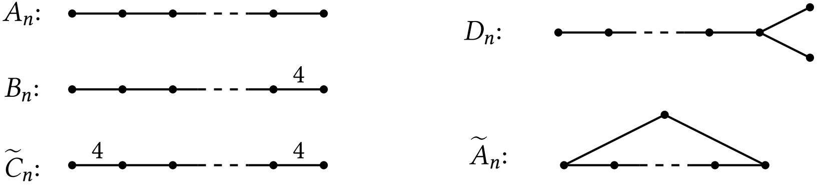


A: They admit geometric embeddings into mapping class groups!

Geometric = standard generators map to Dehn twists or half-twists.

Mapping class groups

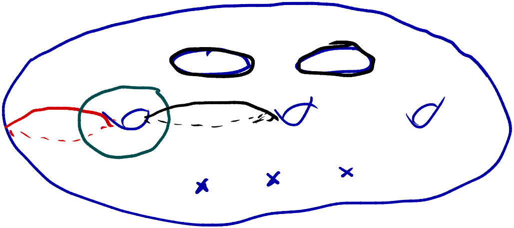
Let $S = S_{g,b}$ be a compact, connected, oriented surface of genus g with $b \geqslant 0$ boundary components, and let $P \subset \mathrm{Int}(S)$ be a finite set of n punctures.

The **mapping class group** of (S, P) is:

$$\mathrm{Mod}(S, \partial S, P) := \pi_0\big(\mathrm{Diff}^+(S;\ \partial S, P)\big)\ ,$$

where $\mathrm{Diff}^+(S;\ \partial S, P)$ is the group of orientation-preserving diffeomorphisms that fix ∂S pointwise and permute P setwise. Isotopies are taken *rel* $\partial S \cup P$.

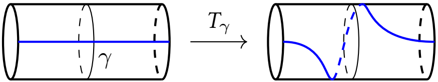
We mod out diffeomorphisms isotopic to identity



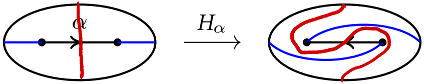
We get a countable group!

Artin relations in mapping class groups

Dehn twists:



Half-twists:



- circles γ, δ / arcs α, β disjoint: $T_\gamma T_\delta = T_\delta T_\gamma, \quad H_\alpha H_\beta = H_\beta H_\alpha$ $(m_{st} = 2)$
- circles γ, δ intersect once / arcs α, β have common endpoint:

$$T_\gamma T_\delta T_\gamma = T_\delta T_\gamma T_\delta, \quad H_\alpha H_\beta H_\alpha = H_\beta H_\alpha H_\beta \qquad (m_{st} = 3)$$

- otherwise: $\langle T_\gamma, T_\delta \rangle \simeq F_2 \simeq \langle H_\alpha, H_\beta \rangle$ $(m_{st} = \infty)$

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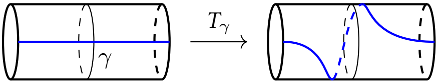

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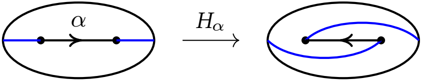

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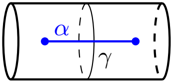
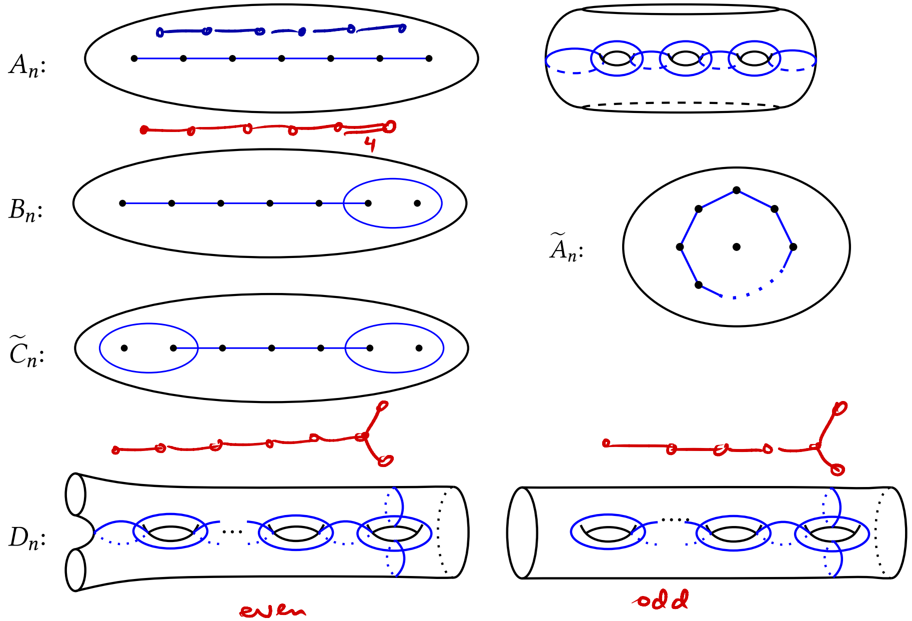
Mixed relation (Labruére–Paris'01):



$$T_\gamma \, H_\alpha \, T_\gamma \, H_\alpha = H_\alpha \, T_\gamma \, H_\alpha \, T_\gamma \qquad (m_{st} = 4)$$

(Almost) all known geometric embeddings:



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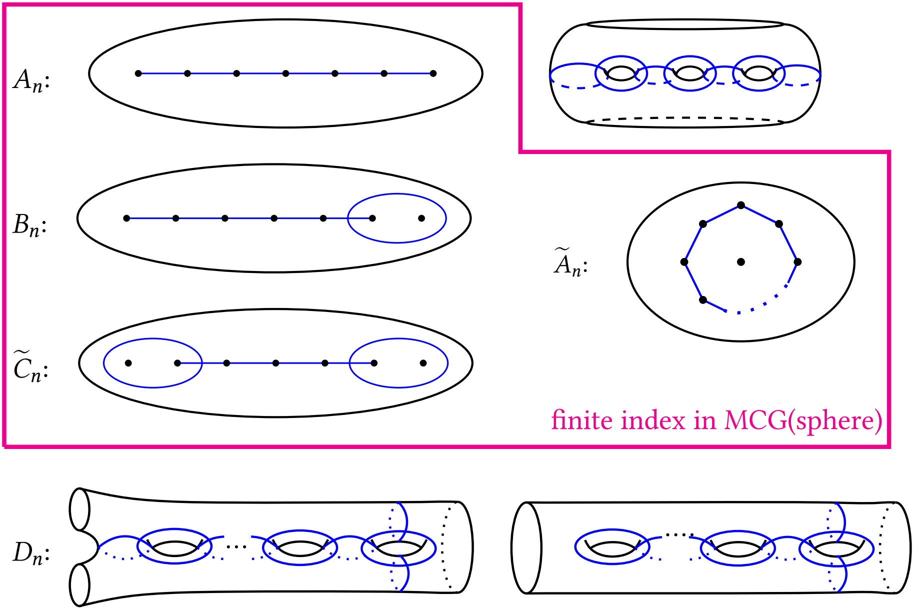

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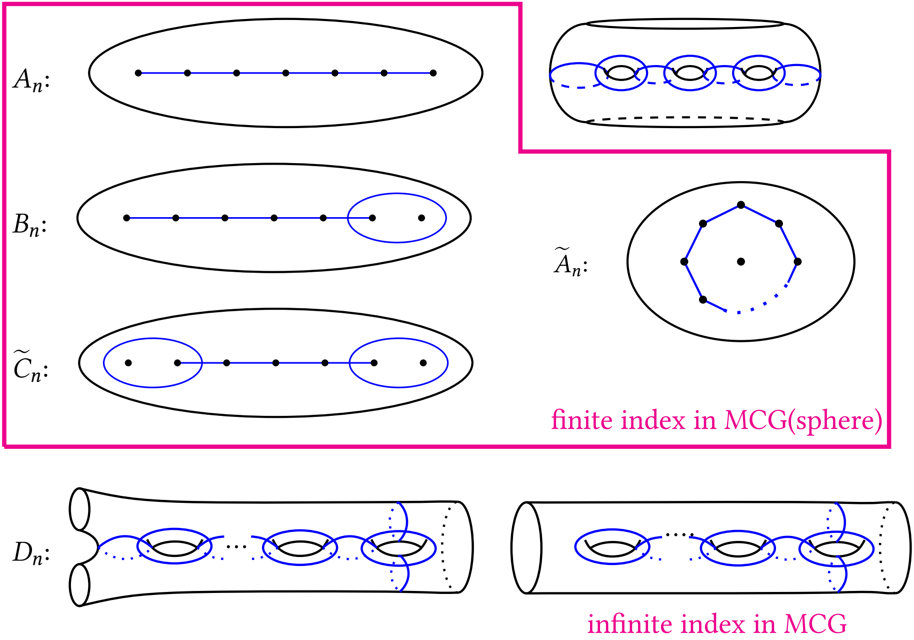


Recent developments

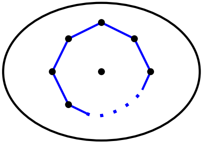
New advances:

- description of homomorphisms of A_n into MCG's, for $n \geqslant 6$ (Castel'16);
- description of homomorphisms from A_n to A_{2n}, for $n \geqslant 4$ (Chen–Kordek–Margalit'19);

allowed obtaining the following new results:

- description of all endomorphisms of D_n, including automorphisms, for $n \geqslant 6$ (Castel–Paris'25+);
- description of all endomorphisms of $\widetilde{A}_n$, for $n \geqslant 4$ (Paris–S.'25);
- description of all endomorphisms of B_n, for $n \geqslant 5$ (Paris–S.'25);
- description of all endomorphisms of $\widetilde{C}_n$, for $n \geqslant 5$ (Paris–S., in preparation);
- R_∞ property for A_n, B_n, $\widetilde{A}_n$, $\widetilde{C}_n$, D_n (Calvez–S.'22, S.–Vaskou'24).

Endomorphisms of $\widetilde{A}_n$:



Theorem (Paris–S.'25)

Let $n \geqslant 4$ and φ be an endomorphism of $\widetilde{A}_n$. Then, up to conjugacy, φ is either

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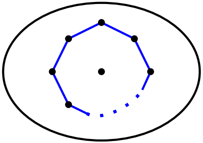

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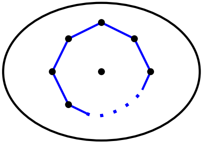
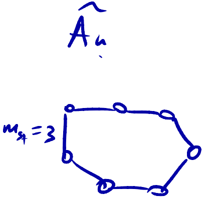
- *cyclic*

Endomorphisms of $\widetilde{A}_n$:

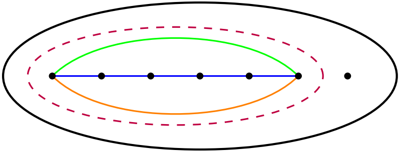




Theorem (Paris–S.'25)

Let $n \geqslant 4$ and φ be an endomorphism of $\widetilde{A}_n$. Then, up to conjugacy, φ is either

- *cyclic, or*
- *an automorphism (description known before by Charney–Crisp'05):*
  (dihedral group of graph autos) $\times$ (global inversion)
- *or non-injective and non-surjective:*



Benefits for mapping class groups:

In “Problems on Mapping Class Groups and Related Topics” (edited by B. Farb, 2006), Joan Birman wrote:

> In a very different direction, every mathematician would do well to have in his or her pile of future projects, in addition to the usual mix, a problem to dream about. In this category I put:
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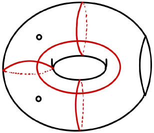
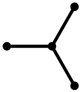
Theorem (S.'20, based on Labruére–Paris'01)

(Pure) MCG $\Bigg\{$  $\Bigg\}$ $\simeq$ Artin(D_4).

D_4 : 

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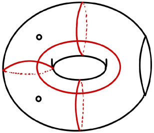
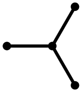
(Pure) MCG $\Bigg\{$  $\Bigg\} \simeq \mathrm{Artin}(D_4)$.

D_4 : 

Corollary. $MCG(g, b, n)$ is linear for

$$(g, b, n) = (1, 3, 0),\ (1, 2, 1),\ (1, 1, 2),\ (1, 0, 3)$$

(since $\mathrm{Artin}(D_4)$ is linear by Digne'03),

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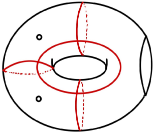
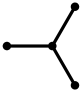
(Pure) MCG $\left\{\right.$  $\left.\right\} \simeq \mathrm{Artin}(D_4)$.

D_4 : 

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(since $\mathrm{Artin}(D_4)$ is linear by Digne’03),

and also for $(g, b, n) = (0, m, n)$, $m \geqslant 2$, $(1,2,0)$, $(1,1,1)$, $(1,0,2)$.

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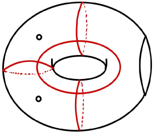
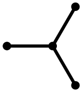
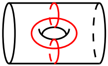
(Pure) MCG $\Bigg\{$  $\Bigg\} \simeq \mathrm{Artin}(D_4)$.

D_4 : 

Corollary. $MCG(g, b, n)$ is linear for

$$(g, b, n) = (1, 3, 0),\ (1, 2, 1),\ (1, 1, 2),\ (1, 0, 3)$$

(since $\mathrm{Artin}(D_4)$ is linear by Digne'03),

and also for $(g, b, n) = (0, m, n),\ m \geqslant 2,\ (1, 2, 0),\ (1, 1, 1),\ (1, 0, 2)$. 

Other applications of geometric embeddings into MCG's:

- Lee–Lee'10: uniqueness of roots up to conjugacy in B_n, $\widetilde{A}_n$, $\widetilde{C}_n$.
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There is some hope for resolving cases (D_6, E_6), (D_7, E_7), (D_8, E_8), but

$$\boldsymbol{(F_4, H_4)}$$

is a really hard nut to crack!

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Motivation: deformations of singularities: Arnold, Milnor, Brieskorn, …

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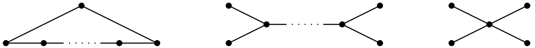

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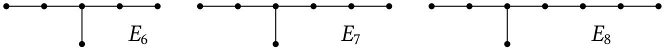

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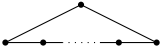
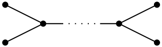
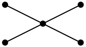
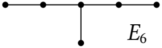
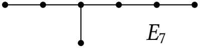
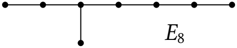
- Labruére'97:   
- Wajnryb'99:  E_6  E_7  E_8

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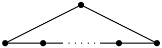
- Mortada'11: REALIZED (!)  (but did not publish)

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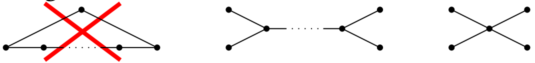
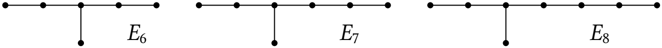
- Labruére'97: 
- Wajnryb'99: 

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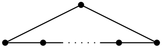
- Mortada'11: REALIZED (!)  (but did not publish)

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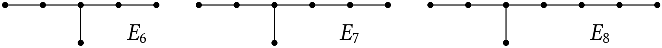
- Wajnryb'99: 

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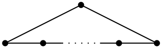
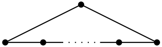
- Mortada'11: REALIZED (!)  (but did not publish)
- Ryffel'23: classified ALL realizations of 

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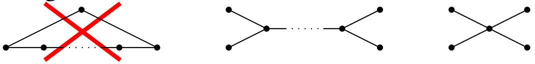
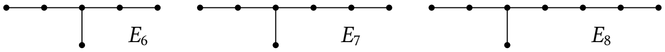
- Labruére'97: 
- Wajnryb'99: 

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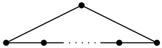
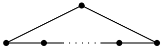
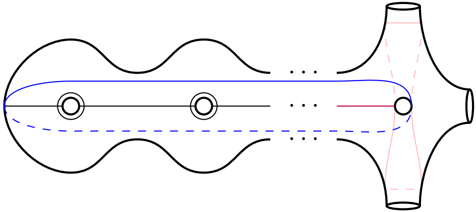
- Mortada'11: REALIZED (!)  (but did not publish)
- Ryffel'23: classified ALL realizations of 



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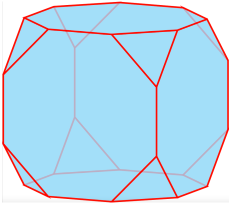

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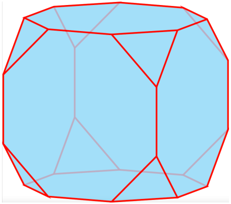
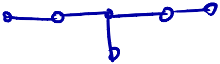
no induced E_6:





There is no obvious obstruction for it to be realizable!

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- I. Soroko, Artin groups of types F_4 and H_4 are not commensurable with that of type D_4, ***Topology and its Applications***, 300 (2021), 107770.
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- L. Paris, I. Soroko, Endomorphisms of Artin groups of type B_n. ***Journal of Algebra***, Volume 678 (2025), 831–861.

Thank you!