Prelim Exam Syllabi (updated 08/2013)

Algebra (includes Linear Algebra)
Linear systems of equations, matrices, rank, inverses, and determinants. Vector spaces, subspaces, linear combinations, spans, linear independence, basis, and dimension. Linear transformations, rank/nullity (dimension) theorem. Coordinates and change of basis. Inner products, Gram-Schmidt, orthogonal projection. Eigenvalues and eigenvectors, diagonalization by change of basis, spectral theorem for real symmetric and Hermitian matrices, orthogonal and unitary matrices, Cayley-Hamilton theorem, Jordan canonical form.
Groups, subgroups and normal subgroups, cyclic and abelian groups, permutation groups, product groups, Lagrange and Cauchy theorems, fundamental theorem of finitely-generated abelian groups, class equation, Sylow theorems, cosets, quotient groups, homomorphisms, isomorphism theorems. Rings, subrings, prime ideals, maximal ideals, principal ideals, quotient rings, ring homomorphisms, integral domains, Fermat's and Euler's theorems, field of fractions, unique factorization domains, principal ideal domains, Euclidean domains, polynomial rings, irreducibility of polynomials over UFDs. Fields, algebraic and transcendental extensions, finite fields, automorphisms of fields, constructability. Splitting fields, separable and normal extensions, Galois theory, insolvability of the quantic.

References:
Kolman & Hill, *Elementary Linear Algebra with Applications*
Lay, *Linear Algebra and Its Applications*, Ch. 1-7
Herstein & Winter, *A Primer on Linear Algebra*
Hoffman & Kunze, *Linear Algebra*
Fraleigh, A *First Course in Abstract Algebra*, 7th Edition, Ch. I-VII, IX, and X
Gallian, *Contemporary Abstract Algebra*, Ch. 1-25, (26), 33
Herstein, *Topics in Algebra*, Ch. 1-6
Garling, *A Course in Galois Theory*

Real Analysis (includes Single- and Multi-variable Calculus)
Axiom of completeness, countable and uncountable sets, uncountability of **R**, convergence of sequences, limits and their properties, monotone convergence theorem, Bolzano-Weierstrass Theorem, Cauchy criterion. Tests for convergence of series, series of functions, absolute and conditional convergence, continuity, uniform continuity and uniform convergence. Topology of R, compactness and connectedness, extreme value theorem, intermediate value theorem, limits at infinity. Differentiation and its properties, implicit differentiation, derivatives of inverse functions, mean value theorem, Darboux Theorem. l'Hospital's Rule, uniform convergence of series of functions, differentiation and integration of power series, Abel's Theorem, Lagrange Remainder Theorem, Taylor series. Riemann integral and its properties, uniform convergence and integration, both Fundamental Theorems of Calculus, Lebesgue criterion for Riemann

integrability, techniques of integration. Fourier series, pointwise convergence of Fourier series, Parseval's equality. Topology of $\mathbf{R}^n$, continuity of functions of several variables, partial derivatives, differentiability of functions of several variables. Tangent space, directional derivatives and gradient, critical points, extrema and Lagrange multipliers, chain rule in several variables, differential of a function. Taylor series in several variables, the inverse and implicit function theorems. Multiple integrals, iterated integrals, change of variables in multiple integrals, polar, cylindrical, spherical coordinates. Integrals over curves and surfaces, Green's Theorem, divergence theorem, Stokes' Theorem.

References:
Understanding Analysis by Stephen Abbott (Springer UTM series), chapters 1-7.
A first course in Real Analysis by Sterling Berberian (also Springer UTM series), chapters 1-9.
Elementary Analysis: the theory of calculus by Ken Ross (UTM series again), chapters 1-6.
Elementary Classical Analysis by Jerrold Marsden (W. H. Freeman and co.), chapters 1-9.
Advanced Calculus of Several Variables by C. H. Edwards, Jr (Dover paperback).
Principles of Mathematical Analysis by Walter Rudin, 3rd edition, whole book.

Complex Analysis
Algebra and geometry of complex numbers, stereographic projection, topology of the complex numbers. Limits and continuity of complex-valued functions, analytic functions, Cauchy-Riemann equations, harmonic functions. Exponential and logarithmic functions, trigonometric functions, linear fractional transformations and their properties, branches of functions. Complex integration, Cauchy's Integral Theorem and Formula, winding numbers, Liouville's Theorem. Taylor series and Laurent series, identity theorem for holomorphic functions, convergence of series of analytic functions, singularities of analytic functions. Residue theorem, application of residues to the evaluation of definite integrals, argument principle, maximum modulus principle, Schwarz Lemma, Rouche's Theorem. Analytic continuation, Schwarz Reflection Principle, conformal mapping, Riemann Mapping Theorem.

References:
An Introduction to Complex Function Theory, by Bruce P. Palka, New York: Springer-Verlag, 1991, Chapters 1-9.

Topology
Basics of point-set topology, including elements of set theory, topological spaces, open and closed sets, bases. Subspace topology, order topology, product topologies, quotient topology, metric topology. Connectedness and compactness, local connectedness and compactness, separation and countability axioms. Classification of surfaces, connected sums, wedge products, Euler characteristic. Pairs of spaces, CW and simplicial complexes, Delta complexes, real and complex projective spaces, cones, suspensions, gluing constructions. Homotopy and homotopy

equivalence, homotopy of paths, fundamental groups, Seifert-van Kampen theorem. Fixed point theorems, covering spaces and their classification, group actions and deck transformations, path lifting, lifting theorem. Simplicial and singular homology, long exact sequence of the pair and triple, Mayer-Vietoris sequences. Homotopy invariance of homology, excision, degree of maps and applications to vector fields. Cellular homology, homology with coefficients, axiomatic approach to homology, relation between fundamental group and H_1 . Basic homological algebra, chain maps, chain homotopies, exact sequences, split exact sequences, five lemma, categories and functors, naturality.

References:
Topology: Second Edition by James R. Munkres, Upper Saddle River, N.J.: Prentice Hall, Inc., 2000. Sections 1-7,12-33.
Algebraic Topology: An Introduction by William S. Massey, New York: Springer-Verlag, 1967, sections 1.1-1.10, 2.1-2.8, 3.1-3.7, 4.1-4.5, 5.1-5.10 .
Algebraic Topology by Allen Hatcher, Cambridge University Press (2001), Chapters 0-2.