

2022 Calculus Bee
Wednesday, April 13, 2022

1st Place	Duc Toan Nguyen
2nd Place	Khoi Nguyen
3rd Place	Brandon Isensee

(1) Find all values of x for which the tangent line to the graph of $y = 505x^2 + x - 2008$ is parallel to the line $y = 2022x + 2008$.

(2) Find

$$\int e^{e^x} e^x dx.$$

(3) Find

$$\int_{-\frac{\pi}{17}}^{\frac{\pi}{17}} \theta^{685} \cos(74\theta) d\theta$$

(4) Evaluate

$$1 + \frac{2}{3} + \frac{2}{3} \cdot \frac{4}{3} + \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{2}{3} \cdot \frac{4}{3} + \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{2}{3} \cdot \frac{4}{3} + \dots$$

(5) Evaluate

$$\int_0^{5/4} \sqrt{25 - 16x^2} dx.$$

(6) Find

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{x^{2022} + 1}}{x^{2022}}.$$

(7) Find $c > 0$ such that the area of the bounded region between $y = c^2 - x^2$ and the x -axis is 1.

(8) Find

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin \theta \cos \theta \tan \theta \sec \theta \cot \theta \csc \theta d\theta.$$

(9) Find r such that the tangent line to $y = x^r$ at $x = 1$ has y -intercept $(0, \frac{1}{3})$.

(10) Evaluate

$$\sum_{n=0}^{\infty} \frac{3^n}{(2n)!}.$$

(11) Find

$$\int_0^3 |x^2 + x - 2| dx.$$

(12) Find the arc length of the curve $y = f(x)$, $e \leq x \leq e^2$, if

$$f(x) = \int_e^x \sqrt{\ln^2(t) - 1} dt.$$

(13) Find

$$\int \frac{t+1}{t^2+1} dt.$$

(14) Suppose that g is a differentiable function such that $\lim_{x \rightarrow \infty} g'(x) = 2022$. Find

$$\lim_{x \rightarrow \infty} \frac{g(x) - 2022}{\sqrt{1 + 2022x^2}}.$$

(15) Evaluate

$$\int (\tan^2 v + \cot^2 v) dv.$$

(16) Evaluate

$$\sum_{n=1}^{\infty} \frac{2^n n^2}{n!}.$$

(17) Find the derivative of

$$F(x) = x^{x^x}.$$

(18) Find

$$\int \frac{z^{87}}{\sqrt{z^{88} + 5}} dz.$$

(19) What is the largest value of a for which $x^3 - ax^2 + ax + 2022$ does not have a local maximum?