Review Exercises

Finding or Evaluating an Integral In Exercises 1–8, use the basic integration rules to find or evaluate the integral.

1. $\int x\sqrt{x^2-36}\,dx$

2. $\int xe^{x^2-1}\,dx$

3. $\int \dfrac{x}{x^2-49}\,dx$

4. $\int \dfrac{x}{\sqrt[3]{4-x^2}}\,dx$

5. $\int_1^e \dfrac{\ln(2x)}{x}\,dx$

6. $\int_{3/2}^2 2x\sqrt{2x-3}\,dx$

7. $\int \dfrac{100}{\sqrt{100-x^2}}\,dx$

8. $\int \dfrac{2x}{x-3}\,dx$

Using Integration by Parts In Exercises 9–16, use integration by parts to find the indefinite integral.

9. $\int xe^{3x}\,dx$

10. $\int x^3 e^x\,dx$

11. $\int e^{2x}\sin 3x\,dx$

12. $\int x\sqrt{x-1}\,dx$

13. $\int x^2 \sin 2x\,dx$

14. $\int \ln\sqrt{x^2-4}\,dx$

15. $\int x\arcsin 2x\,dx$

16. $\int \arctan 2x\,dx$

Finding a Trigonometric Integral In Exercises 17–22, find the trigonometric integral.

17. $\int \cos^3(\pi x-1)\,dx$

18. $\int \sin^2\dfrac{\pi x}{2}\,dx$

19. $\int \sec^4\dfrac{x}{2}\,dx$

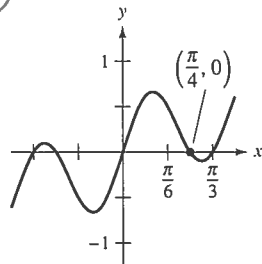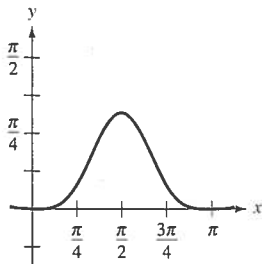20. $\int \tan\theta\sec^4\theta\,d\theta$

21. $\int \dfrac{1}{1-\sin\theta}\,d\theta$

22. $\int \cos 2\theta(\sin\theta+\cos\theta)^2\,d\theta$

Area In Exercises 23 and 24, find the area of the region.

23. $y=\sin^4 x$

24. $y=\sin 3x\cos 2x$

Using Trigonometric Substitution In Exercises 25–30, use trigonometric substitution to find or evaluate the integral.

25. $\int \dfrac{-12}{x^2\sqrt{4-x^2}}\,dx$

26. $\int \dfrac{\sqrt{x^2-9}}{x}\,dx,\quad x>3$

27. $\int \dfrac{x^3}{\sqrt{4+x^2}}\,dx$

28. $\int \sqrt{25-9x^2}\,dx$

29. $\int_0^1 \dfrac{6x^3}{\sqrt{16+x^2}}\,dx$

30. $\int_3^4 x^3\sqrt{x^2-9}\,dx$

Using Different Methods In Exercises 31 and 32, find the indefinite integral using each method.

31. $\int \dfrac{x^3}{\sqrt{4+x^2}}\,dx$

  (a) Trigonometric substitution

  (b) Substitution: $u^2=4+x^2$

  (c) Integration by parts: $dv=\dfrac{x}{\sqrt{4+x^2}}\,dx$

32. $\int x\sqrt{4+x}\,dx$

  (a) Trigonometric substitution

  (b) Substitution: $u^2=4+x$

  (c) Substitution: $u=4+x$

  (d) Integration by parts: $dv=\sqrt{4+x}\,dx$

Using Partial Fractions In Exercises 33–38, use partial fractions to find the indefinite integral.

33. $\int \dfrac{x-39}{x^2-x-12}\,dx$

34. $\int \dfrac{5x-2}{x^2-x}\,dx$

35. $\int \dfrac{x^2+2x}{x^3-x^2+x-1}\,dx$

36. $\int \dfrac{4x-2}{3(x-1)^2}\,dx$

37. $\int \dfrac{x^2}{x^2+5x-24}\,dx$

38. $\int \dfrac{\sec^2\theta}{\tan\theta(\tan\theta-1)}\,d\theta$

Integration by Tables In Exercises 39–46, use integration tables to find or evaluate the integral.

39. $\int \dfrac{x}{(4+5x)^2}\,dx$

40. $\int \dfrac{x}{\sqrt{4+5x}}\,dx$

41. $\int_0^{\sqrt{\pi}/2} \dfrac{x}{1+\sin x^2}\,dx$

42. $\int_0^1 \dfrac{x}{1+e^{x^2}}\,dx$

43. $\int \dfrac{x}{x^2+4x+8}\,dx$

44. $\int \dfrac{3}{2x\sqrt{9x^2-1}}\,dx,\quad x>\dfrac{1}{3}$

45. $\int \dfrac{1}{\sin\pi x\cos\pi x}\,dx$

46. $\int \dfrac{1}{1+\tan\pi x}\,dx$

47. **Verifying a Formula** Verify the reduction formula
$$\int (\ln x)^n\,dx = x(\ln x)^n - n\int (\ln x)^{n-1}\,dx.$$

48. **Verifying a Formula** Verify the reduction formula
$$\int \tan^n x\,dx = \dfrac{1}{n-1}\tan^{n-1}x - \int \tan^{n-2}x\,dx.$$

Finding an Indefinite Integral In Exercises 49–56, find the indefinite integral using any method.

49. $\displaystyle\int \theta \sin\theta\cos\theta\, d\theta$

50. $\displaystyle\int \frac{\csc\sqrt{2x}}{\sqrt{x}}\, dx$

51. $\displaystyle\int \frac{x^{1/4}}{1+x^{1/2}}\, dx$

52. $\displaystyle\int \sqrt{1+\sqrt{x}}\, dx$

53. $\displaystyle\int \sqrt{1+\cos x}\, dx$

54. $\displaystyle\int \frac{3x^3+4x}{(x^2+1)^2}\, dx$

55. $\displaystyle\int \cos x\,\ln(\sin x)\, dx$

**56.** $\displaystyle\int (\sin\theta+\cos\theta)^2\, d\theta$

Differential Equation In Exercises 57–60, solve the differential equation using any method.

57. $\dfrac{dy}{dx}=\dfrac{25}{x^2-25}$

58. $\dfrac{dy}{dx}=\dfrac{\sqrt{4-x^2}}{2x}$

59. $y'=\ln(x^2+x)$

60. $y'=\sqrt{1-\cos\theta}$

Evaluating a Definite Integral In Exercises 61–66, evaluate the definite integral using any method. Use a graphing utility to verify your result.

61. $\displaystyle\int_2^{\sqrt{5}} x(x^2-4)^{3/2}\, dx$

62. $\displaystyle\int_0^1 \frac{x}{(x-2)(x-4)}\, dx$

63. $\displaystyle\int_1^4 \frac{\ln x}{x}\, dx$

64. $\displaystyle\int_0^2 xe^{3x}\, dx$

65. $\displaystyle\int_0^{\pi} x\sin x\, dx$

66. $\displaystyle\int_0^5 \frac{x}{\sqrt{4+x}}\, dx$

Area In Exercises 67 and 68, find the area of the region.

67. $y=x\sqrt{4-x}$

68. $y=\dfrac{1}{25-x^2}$
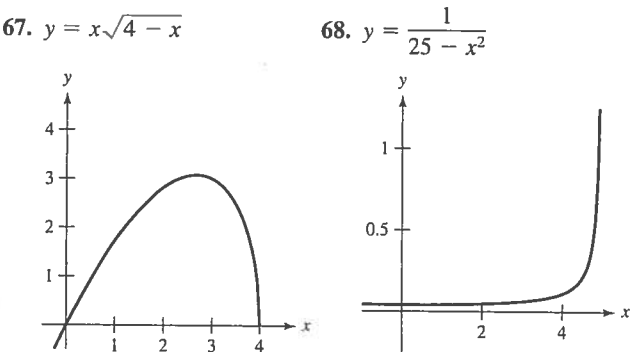
Centroid In Exercises 69 and 70, find the centroid of the region bounded by the graphs of the equations.

69. $y=\sqrt{1-x^2}$, $y=0$

70. $(x-1)^2+y^2=1$, $(x-4)^2+y^2=4$

Arc Length In Exercises 71 and 72, approximate to two decimal places the arc length of the curve over the given interval.

Function	Interval
71. $y=\sin x$	$[0,\pi]$
72. $y=\sin^2 x$	$[0,\pi]$

Evaluating a Limit In Exercises 73–80, use L'Hôpital's Rule to evaluate the limit.

73. $\displaystyle\lim_{x\to 1}\frac{(\ln x)^2}{x-1}$

74. $\displaystyle\lim_{x\to 0}\frac{\sin\pi x}{\sin 5\pi x}$

75. $\displaystyle\lim_{x\to\infty}\frac{e^{2x}}{x^2}$

76. $\displaystyle\lim_{x\to\infty} xe^{-x^2}$

77. $\displaystyle\lim_{x\to\infty}(\ln x)^{2/x}$

78. $\displaystyle\lim_{x\to 1^+}(x-1)^{\ln x}$

79. $\displaystyle\lim_{n\to\infty}1000\left(1+\frac{0.09}{n}\right)^n$

80. $\displaystyle\lim_{x\to 1^+}\left(\frac{2}{\ln x}-\frac{2}{x-1}\right)$

Evaluating an Improper Integral In Exercises 81–88, determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

81. $\displaystyle\int_0^{16}\frac{1}{\sqrt[4]{x}}\, dx$

82. $\displaystyle\int_0^2 \frac{7}{x-2}\, dx$

83. $\displaystyle\int_1^{\infty} x^2\ln x\, dx$

84. $\displaystyle\int_0^{\infty}\frac{e^{-1/x}}{x^2}\, dx$

85. $\displaystyle\int_1^{\infty}\frac{\ln x}{x^2}\, dx$

86. $\displaystyle\int_1^{\infty}\frac{1}{\sqrt[4]{x}}\, dx$

87. $\displaystyle\int_2^{\infty}\frac{1}{x\sqrt{x^2-4}}\, dx$

88. $\displaystyle\int_0^{\infty}\frac{2}{\sqrt{x}(x+4)}\, dx$

89. Present Value The board of directors of a corporation is calculating the price to pay for a business that is forecast to yield a continuous flow of profit of \$500,000 per year. The money will earn a nominal rate of 5% per year compounded continuously. What is the present value of the business

(a) for 20 years?

(b) forever (in perpetuity)?

(*Note:* The present value for t_0 years is $\int_0^{t_0} 500{,}000e^{-0.05t}\, dt$.)

90. Volume Find the volume of the solid generated by revolving the region bounded by the graphs of $y=xe^{-x}$, $y=0$, and $x=0$ about the x-axis.

91. Probability The average lengths (from beak to tail) of different species of warblers in the eastern United States are approximately normally distributed with a mean of 12.9 centimeters and a standard deviation of 0.95 centimeter (see figure). The probability that a randomly selected warbler has a length between a and b centimeters is

$$P(a\le x\le b)=\frac{1}{0.95\sqrt{2\pi}}\int_a^b e^{-(x-12.9)^2/1.805}\, dx.$$

Use a graphing utility to approximate the probability that a randomly selected warbler has a length of (a) 13 centimeters or greater and (b) 15 centimeters or greater. (*Source: Peterson's Field Guide: Eastern Birds*)