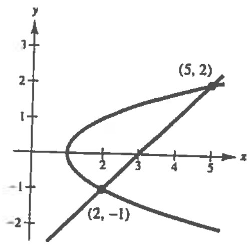
Solutions to review problems for Test 1.

6. $A = \int_{-1}^{2} \left[(y+3) - (y^2+1)\right] dy$

$= \int_{-1}^{2} (2 + y - y^2)\, dy = \left[2y + \frac{1}{2}y^2 - \frac{1}{3}y^3\right]_{-1}^{2} = \frac{9}{2}$

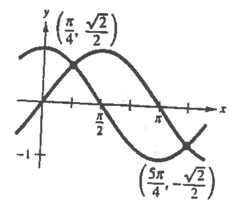


9. $A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx$

$= \left[-\cos x - \sin x\right]_{\pi/4}^{5\pi/4}$

$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$

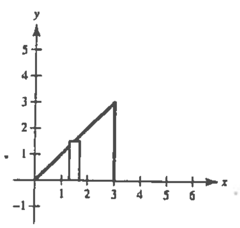
$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$



---

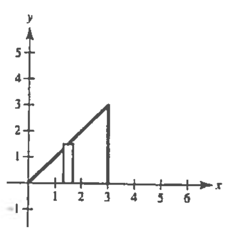
17. (a) **Disk**

$V = \pi \int_0^3 x^2\, dx = \left[\frac{\pi x^3}{3}\right]_0^3 = 9\pi$



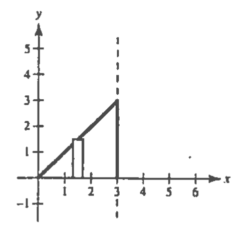
(b) **Shell**

$V = 2\pi \int_0^3 x(x)\, dx = 2\pi \left[\frac{x^3}{3}\right]_0^3 = 18\pi$



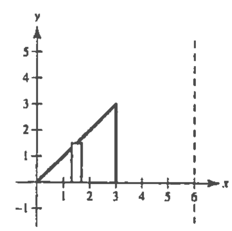
(c) **Shell**

$V = 2\pi \int_0^3 (3 - x)x\, dx = 2\pi \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = 9\pi$



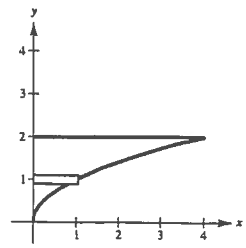
(d) **Shell**

$V = 2\pi \int_0^3 (6 - x)x\, dx = 2\pi \left[3x^2 - \frac{x^3}{3}\right]_0^3 = 36\pi$



18. (a) **Shell**

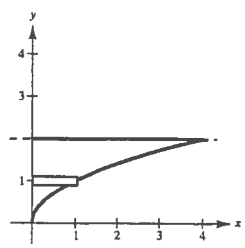
$$V = 2\pi\int_0^2 y^3\,dy = \left[\frac{\pi}{2}y^4\right]_0^2 = 8\pi$$



(b) **Shell**

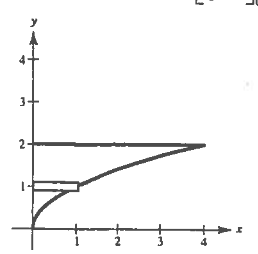
$$V = 2\pi\int_0^2 (2-y)y^2\,dy$$

$$= 2\pi\int_0^2 \left(2y^2 - y^3\right)dy = 2\pi\left[\frac{2}{3}y^3 - \frac{1}{4}y^4\right]_0^2 = \frac{8\pi}{3}$$



(c) **Disk**

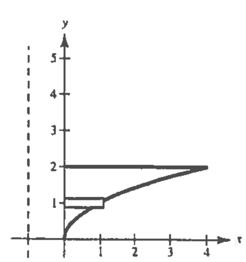
$$V = \pi\int_0^2 y^4\,dy = \left[\frac{\pi}{5}y^5\right]_0^2 = \frac{32\pi}{5}$$



(d) **Disk**

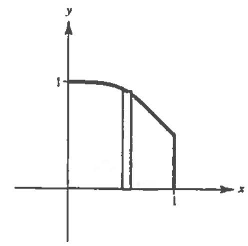
$$V = \pi\int_0^2\left[\left(y^2+1\right)^2 - 1^2\right]dy$$

$$= \pi\int_0^2\left(y^4 + 2y^2\right)dy = \pi\left[\frac{1}{5}y^5 + \frac{2}{3}y^3\right]_0^2 = \frac{176\pi}{15}$$



19. Shell

$$V = 2\pi\int_0^1 \frac{x}{x^4+1}\,dx = \pi\int_0^1 \frac{(2x)}{\left(x^2\right)^2+1}\,dx = \left[\pi\arctan\left(x^2\right)\right]_0^1 = \pi\left(\frac{\pi}{4} - 0\right) = \frac{\pi^2}{4}$$



21. Shell

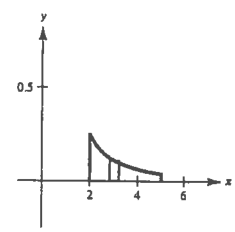
$$V = 2\pi\int_2^5 x\left(\frac{1}{x^2}\right)dx$$

$$= 2\pi\int_2^5 \frac{1}{x}\,dx$$

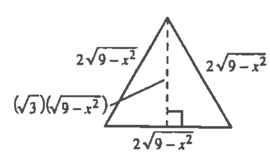
$$= \left[2\pi\ln|x|\right]_2^5$$

$$= 2\pi(\ln 5 - \ln 2)$$

$$= 2\pi\ln\left(\frac{5}{2}\right)$$



24.



$$A(x) = \frac{1}{2}bh = \frac{1}{2}\left(2\sqrt{9-x^2}\right)\left(\sqrt{3}\sqrt{9-x^2}\right)$$

$$= \sqrt{3}\left(9 - x^2\right)$$

$$V = \sqrt{3}\int_{-3}^3\left(9 - x^2\right)dx = \sqrt{3}\left[9x - \frac{x^3}{3}\right]_{-3}^3$$

$$= \sqrt{3}\left[(27-9) - (-27+9)\right] = 36\sqrt{3}$$

25.
$$f(x) = \tfrac{4}{5}x^{5/4}$$
$$f'(x) = x^{1/4}$$
$$1 + [f'(x)]^2 = 1 + \sqrt{x}$$
$$u = 1 + \sqrt{x}$$
$$x = (u-1)^2$$
$$dx = 2(u-1)\,du$$
$$s = \int_0^4 \sqrt{1+\sqrt{x}}\,dx = 2\int_1^3 \sqrt{u}(u-1)\,du$$
$$= 2\int_1^3 \left(u^{3/2} - u^{1/2}\right) du$$
$$= 2\left[\tfrac{2}{5}u^{5/2} - \tfrac{2}{3}u^{3/2}\right]_1^3 = \tfrac{4}{15}\left[u^{3/2}(3u-5)\right]_1^3$$
$$= \tfrac{8}{15}\left(1 + 6\sqrt{3}\right) = 6.076$$

26.
$$y = \frac{x^3}{6} + \frac{1}{2x}$$
$$y' = \frac{1}{2}x^2 - \frac{1}{2x^2}$$
$$1 + (y')^2 = \left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right)^2$$
$$s = \int_1^3 \left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right) dx = \left[\frac{1}{6}x^3 - \frac{1}{2x}\right]_1^3 = \frac{14}{3}$$

30. $y = 2\sqrt{x},\ y' = \dfrac{1}{\sqrt{x}}$
$$1 + (y')^2 = 1 + \frac{1}{x} = \frac{x+1}{x}$$
$$S = 2\pi\int_3^8 2\sqrt{x}\sqrt{\frac{x+1}{x}}\,dx = 4\pi\int_3^8 \sqrt{x+1}\,dx$$
$$= 4\pi\left[\frac{2}{3}(x+1)^{3/2}\right]_3^8 = \frac{152\pi}{3}$$

32. $F = kx$
$$50 = k(1) \Rightarrow k = 50$$
$$W = \int_0^{10} 50x\,dx = \left[25x^2\right]_0^{10} = 2500 \text{ in-lb} \approx 208.3 \text{ ft-lb}$$

33. Volume of disk: $\pi\left(\dfrac{1}{3}\right)^2 \Delta y$ $\quad\left[\text{diameter} = \dfrac{2}{3}\text{ ft}\right]$

Weight of disk: $62.4\pi\left(\dfrac{1}{3}\right)^2 \Delta y$

Distance: $190 - y$
$$W = \frac{62.4\pi}{9}\int_0^{165}(190 - y)\,dy$$
$$= \frac{62.4\pi}{9}\left[190y - \frac{y^2}{2}\right]_0^{165}$$
$$= \frac{62.4\pi}{9}\left[\frac{35,475}{2}\right] = 122,980\pi \text{ ft-lb}$$
$$\approx 193.2 \text{ foot-tons}$$