$g(x) = \tan x$.

g is one-to-one since if $x_1 \neq x_2$ (say $x_1 > x_2$) $\Rightarrow g(x_1) > g(x_2)$, i.e. $g(x_1) \neq g(x_2)$ (g is increasing)

g is onto, since $\forall y \in \mathbb{R}$ $\exists$ $x = \arctan y \in (-\pi/2, \pi/2)$ s.t. $g(x) = y$.

Then, from problem 1 $h = g \circ f : (0,1) \to \mathbb{R}$ is one-to-one and onto.

#3. Let $C = B \setminus A$. Then $A \cup B = A \cup C$ and $A \cap C = \emptyset$.

- If $C = \emptyset$, then $A \cup B = A$ is countable.
- If $C \neq \emptyset \Rightarrow C$ is finite. Let n be the number of elements in C, i.e.

$C = \{c_1, c_2, \ldots, c_n\}$. Since A is countable $\exists$ $f: \mathbb{N} \to A$, i.e. one can list $a_1 = f(1), a_2 = f(2), \ldots$

Define $g: \mathbb{N} \to A \cup C$ by

$$g(k) = \begin{cases} c_k, & 1 \le k \le n \\ f(k-n) = a_{k-n}, & \text{if } k > n. \end{cases}$$

Then g is one-to-one and onto, thus $A \cup C$ is countable.

#1.5.6 a) It is possible: $\{(n, n+1), n \in \mathbb{N}\}$

b) Not possible: from theorem 1.4.3 $\Rightarrow$ each (a,b) contains a rational number. Thus existence of an uncountable collection of distinct open intervals would imply existence of uncountable collection of distinct rational numbers.