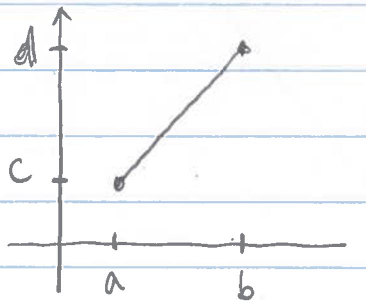
Homework # 9 (Solutions)

#1. <u>h is one-to-one</u>: $\overbrace{\text{If } h(x) = h(y)}^{\text{For any } x, y \in A}$, then

$g(f(x)) = g(f(y))$. Since g is one-to-one $\Rightarrow$
$f(x) = f(y)$, and since f is one-to-one $\Rightarrow$ $x = y$
So h is one-to-one by definition.

<u>h is onto</u>: Given any $c \in C$, one can find $b \in B$
s.t. $g(b) = c$; since g is onto, one can find
$a \in A$ s.t. $f(a) = b$, since f is onto. Thus $h(a) = g(f(a)) = c$.

#2.



a) Let f be a linear function passing through points (a, c) and (b, d), i.e.

$$f(x) - c = \frac{d-c}{b-a}(x-a).$$

To check that f is one-to-one, observe that

$$f(x_1) = f(x_2) \Rightarrow \frac{d-c}{b-a}(x_1 - a) = \frac{d-c}{b-a}(x_2 - a) \Rightarrow x_1 = x_2.$$

To check that f is onto, let $c \le y \le d$,
solve $y - c = \frac{d-c}{b-a}(x-a)$ for x:

$x = \frac{b-a}{d-c}(y-c) + a$. Then $x \ge \frac{b-a}{d-c}(c-c) + a = a$,
and $x \le \frac{b-a}{d-c}(d-c) + a = b$. Thus, given $y \in [c, d]$,
$\exists\, x \in [a, b]$, s.t. $f(x) = y$.

b) Let $f : (0, 1) \to (-\pi/2, \pi/2)$ be

$f(x) = \pi x - \pi/2$, $f(x)$ is one-to-one and onto from a). Let $g : (-\pi/2, \pi/2) \to \mathbb{R}$,