

(c) Each I_n is open, no two I_n and I_m are equal, and

$$\bigcap_{n=1}^{\infty} I_n = [3, 4].$$

2. Prove that between any two real numbers $a < b$ there are infinitely many rational numbers. (You can use Thm 1.4.3).

3. Let a be any arbitrary real number.

Find $\sup \{r \in \mathbb{Q} \mid r < a\}$ and prove your answer.

4. Use the Archimedean principle to prove that if $A_n = [0, \frac{1}{n}]$ and $B_n = [n, +\infty)$, then $\bigcap_{n=1}^{\infty} A_n = \{0\}$ and $\bigcap_{n=1}^{\infty} B_n = \emptyset$.