Homework #8 (Due 09/10/2018)

- Read pp. 20–26.

- Need to know: Statements and proofs of Lemma 1.3.8, theorem 1.4.1, theorem 1.4.2, theorem 1.4.3, and corollary 1.4.4.

- You may redo problem #1.3.3 for full credit by completing the following steps:

  1. Apply Axiom of Completeness to the set B and conclude that $\sup B$ exists.

  2. Explain why $\sup B$ is a lower bound of A.

  3. Explain why $\sup B$ is the greatest lower bound of A and thus $\inf A$ exists and is equal to $\sup B$.

  4. Explain why it is sufficient to have Axiom of Completeness for supremum and one does not need an axiom that guarantees the existence of infimum.

- Do the following problems:

1. In each case find a nested collection $I_1 \supseteq I_2 \supseteq \dots$ of intervals as described

(a) Each I_n is open, i.e. $I_n = (a_n, b_n)$, $a_n < b_n$ and $\bigcap_{n=1}^{\infty} I_n = \{3\}$.

(b) Each I_n is closed, i.e. $I_n = [a_n, b_n]$, $a_n \leq b_n$, no two I_n and I_m are equal, and $\bigcap_{n=1}^{\infty} I_n = [3, 4]$.