Homework # 5 (Solutions)

1.3.1 a) A real number m is the greatest lower bound (or infimum) of a set $A \subseteq \mathbb{R}$, if

i) m is a lower bound of A;

ii) if ℓ is any lower bound of A, then $m \geq \ell$.

1.3.2 a) This is possible: if $B = \{1\}$, then $\inf(B) = 1 \geq \sup(B) = 1$.

b) This is not possible. If set A is finite with n elements, one can arrange elements of A as $a_1 < a_2 < \ldots < a_n$. Then $\inf(A) = a_1 \in A$ and $\sup(A) = a_n \in A$.

c) Let $C = (0, 1] \cap \mathbb{Q}$, then $\sup C = 1 \in C$, $\inf C = 0 \notin C$.

#2. a) $B = \{n \in \mathbb{N} \mid n^2 < 17\} = \{1, 2, 3, 4\}$.
Thus $\sup B = 4$ and $\inf B = 1$.

b) Observe that $\frac{1}{2} < \frac{n}{2n-1} \leq 1$ and that

$\frac{n}{2n-1} = 1$ when $n = 1$ and $\lim\limits_{n \to \infty} \frac{n}{2n-1} = \frac{1}{2}$.

Thus $\sup B = 1$ and $\inf B = \frac{1}{2}$.

c) If $m, n \in \mathbb{N}$, $m+n < 12$, then $\max B = \sup B = \frac{10}{1}$ and $\min B = \inf B = \frac{1}{10}$.

d) Observe that $0 < \frac{m}{n+m} < 1$ with $\lim\limits_{m \to \infty} \frac{m}{n+m} = 1$ and $\lim\limits_{n \to \infty} \frac{m}{n+m} = 0$. Thus $\sup(B) = 1$ and $\inf(B) = 0$.

e) $\sup(B) = \sqrt{17}$. $\inf(B) = -\sqrt{17}$.