# Solutions to homework #3

1. 2.9 (a) $f^{-1}([0,4)) = [-2,2]$ and $f^{-1}([-1,1]) = [-1,1]$.

So $f^{-1}(A \cap B) = f^{-1}([0,1]) = [-1,1] = f^{-1}(A) \cap f^{-1}(B)$,

$f^{-1}(A \cup B) = f^{-1}([-1,4]) = [-2,2] = f^{-1}(A) \cup f^{-1}(B)$.

(b) $x \in g^{-1}(A \cap B) \iff g(x) \in A \cap B \iff g(x) \in A$ and $g(x) \in B \iff x \in g^{-1}(A)$ and $x \in g^{-1}(B) \iff x \in g^{-1}(A) \cap g^{-1}(B)$.

So we proved that

$$g^{-1}(A \cap B) = g^{-1}(A) \cap g^{-1}(B)$$

$x \in g^{-1}(A \cup B) \iff g(x) \in A \cup B \iff g(x) \in A$ or $g(x) \in B \iff x \in g^{-1}(A)$ or $x \in g^{-1}(B) \iff x \in g^{-1}(A) \cup g^{-1}(B)$

So we proved that

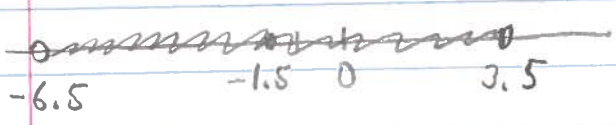
$$g^{-1}(A \cup B) = g^{-1}(A) \cup g^{-1}(B).$$

#2

a) $|2x+3| = 2|x+3/2| < 10$

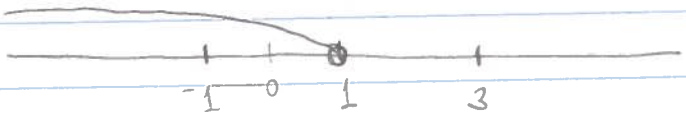
$\Rightarrow |x+3/2| < 5$.

Distance from $x$ to $-3/2$ is less than 5




Answer: $x \in (-6.5, 3.5)$

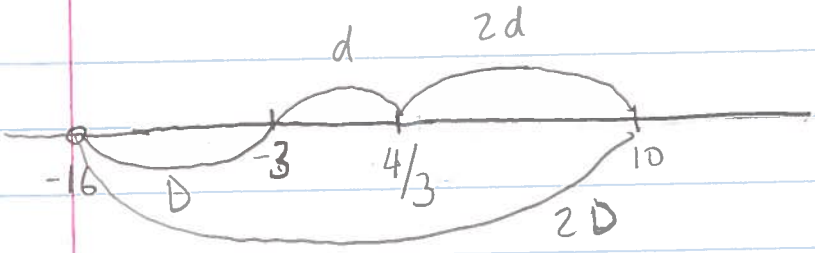
b) Distance from $x$ to 3 is greater than the distance from $x$ to $-1$.




Answer: $(-\infty, 1)$

c) $|2x+6| \leq |x-10|$ or $|x+3| \leq \frac{1}{2}|x-10|$

Distance from $x$ to $-3$ is less or equal than half of the distance from $x$ to 10




Answer: $(-16, 4/3)$