

Homework # 29.

- Read pp. 148 - 158.
- Need to know: Thm (5.2.5, Chain rule, with proof), Thm 5.2.6 (with proof), Thm 5.2.7, Thm 5.3.1 (Rolle's Theorem, with proof), Thm. 5.3.2 (with proof)

Do the following problems:

1. Suppose $f: \mathbb{R} \rightarrow \mathbb{R}$ is differentiable on \mathbb{R} , and $f'(x)$ is continuous on \mathbb{R} .

a) Let $[a, b]$ be arbitrary, argue that $\exists M > 0$ such that $|f'(x)| \leq M, \forall x \in [a, b]$.

b) Use the Mean value theorem to show that $\forall x, y \in [a, b], x \neq y, |f(x) - f(y)| \leq M|x - y|$.

c) Suppose that $f'(x)$ is bounded on \mathbb{R} , prove that $f(x)$ is uniformly continuous on \mathbb{R} .

d) Prove that $f(x)$ is increasing on $\mathbb{R} \iff f'(x) > 0$ on \mathbb{R} .

Do # 5.3.2, 5.3.3