Homework # 22 (Solutions)

3.3.4 a) $K \cap F$ is always compact: $K \cap F \subseteq K \Rightarrow$ $K \cap F$ is bounded. Also $K \cap F$ is closed since both K and F are closed. Thus $K \cap F$ is compact.

b) Closure of any set is closed $\Rightarrow$ $\overline{F^c \cup K^c}$ is $\underbrace{\text{closed}}_{\text{always}}$. Let $F = K = [0,1]$, then $F^c \cup K^c = (-\infty, 0) \cup (1, +\infty)$ and $\overline{F^c \cup K^c}$ $= (-\infty, 0] \cup [1, +\infty)$ is not bounded. Thus $\overline{F^c \cup K^c}$ may not be compact.

c) If $K = [0,1]$ and $F = [0, \frac{1}{2}]$, then $K \setminus F = (\frac{1}{2}, 1)$ is not closed $\Rightarrow$ also not compact.

d) Since $\overline{K \cap F^c} \subseteq K \Rightarrow \overline{K \cap F^c}$ is bounded. $\overline{K \cap F^c}$ is also closed $\Rightarrow \overline{K \cap F^c}$ is compact.

3.3.5. a) True, Let K_λ be all compact, $\lambda \in \Lambda$. Then $\bigcap_{\lambda \in \Lambda} K_\lambda$ is closed, since all K_λ are closed.

$\bigcap_{\lambda \in \Lambda} K_\lambda$ is bounded, since $\bigcap_{\lambda \in \Lambda} K_\lambda \subseteq K_{\lambda'}$ $\forall \lambda' \in \Lambda$.

$\Rightarrow \bigcap_{\lambda \in \Lambda} K_\lambda$ is compact.

(b) <u>False</u>. Sets $K_n = \{n\}$ are compact.

$\bigcup_{n=1}^{\infty} K_n = \mathbb{N}$ is not bdd $\Rightarrow$ not cpct.

(c) <u>False</u> $A = (0,1)$; $K = [0,1]$

Set $A \cap K = (0,1)$ is not closed $\Rightarrow$ not cpct.

(d) <u>False</u>. Let $A_n = [n, +\infty)$. A_n is closed $\forall n \in \mathbb{N}$, yet $\bigcap_{n=1}^{\infty} A_n = \emptyset$.