Homework #19.

· Read pp. 85 – 93.

Need to know: Definitions of open and closed sets, limit point. Statements and proofs of Thm 3.2.3, 3.2.4, 3.2.8.

Do the following problems:

1. Prove that if there is a sequence (a_n) in A such that $\lim a_n = X$, then X is a limit point of A.

2. Let $A = \left\{ \dfrac{n \cdot \sin\left(\frac{n\pi}{4}\right)}{n+1} : n = 1, 2, 3, 4, \ldots \right\}$

(a) What are the limit points.
(b) Does A contain any isolated points.
(c) Find the closure of A.

3. Decide if the following sets are open, closed, or neither. Justify your answers.

a) $\left\{ \dfrac{\sin\left(\frac{n\pi}{2}\right)}{n} : n = 1, 2, 3, \ldots \right\}$;

b) $\bigcup_{n=1}^{\infty} [1 - 1/n, +\infty)$;

c) $\mathbb{R}$;

d) $\mathbb{Q} \cap [0, 1]$.

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4. Determine if the following series converges or diverges. If converges, does it converge absolutely. Provide justification.

a) $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n})$;

b) $\sum_{n=1}^{\infty} e^{-n^2}$;

c) $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right)$;

d) $\sum_{n=1}^{\infty} \frac{2^n + 4n}{3^n + 5^n}$;

e) $\sum_{n=1}^{\infty} (-1)^n \frac{n^{1000}}{(1.01)^n}$

5. Do 3.2.6.