

## Homework # 18.

- Read pp. 71-76.

Do the following problems.

1. Let  $(a_k)$  and  $(b_k)$  be sequences satisfying  $0 \leq a_k \leq b_k \quad \forall k \in \mathbb{N}$ .

(a) Prove that if  $\sum_{k=1}^{\infty} b_k$  converges, then  $\sum_{k=1}^{\infty} a_k$  converges.

(b) Prove that if  $\sum_{k=1}^{\infty} a_k$  diverges, then  $\sum_{k=1}^{\infty} b_k$  diverges.

2. Prove that if the series  $\sum_{n=1}^{\infty} |a_n|$  converges, then  $\sum_{n=1}^{\infty} a_n$  converges.

3. Do # 2.7.9.

4. Determine if the following series converge or diverge. Provide brief justification

(a)  $\sum_{n=1}^{\infty} \frac{3n}{n^3+1}$  ; (b)  $\sum_{n=1}^{\infty} \frac{(-1)^n \log n}{n}$

(c)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[n]{n}}$  ; (d)  $\sum_{n=1}^{\infty} \frac{2^n}{n!}$