Homework # 16 (Solutions)

1. Since (a_n) is unbounded, there is no upper bound or lower bound. Without loss of generality, we assume that (a_n) does not have an upper bound. Choose n_1 so that $a_{n_1} \geq 1$, choose $n_2 > n_1$ such that $a_{n_2} \geq 2$, ..., choose $n_k > n_{k-1}$ such that $a_{n_k} \geq k$, ...
This can be done since $\forall M$, there are infinitely many terms in (a_n) that are greater than M. Thus, $\lim_{k\to\infty} a_{n_k} \geq \lim_{k\to\infty} k = +\infty$.

#2. Since $\lim_{n\to\infty} a_n = +\infty$, $\forall M>0$ $\exists N(M)$ s.t. $\forall n \geq N(M)$, $a_n > M$.
Then $\forall k \geq N(M)$, n_k is also $\geq N(M)$, so $a_{n_k} > M$. Thus we checked the definition that $\lim_{k\to\infty} a_{n_k} = +\infty$.

2.5.5. (Done in hwk #15)

#4. We will do a general case of #4 in 2.7.1(c) in hwk 17.