Homework #16

- Read pp. 62–67.

- Need to know statement and proof of Bolzano-Weierstrass theorem. Definition of Cauchy sequence.

Do the following problems:

1. Suppose that sequence (a_n) is unbounded. Prove that there is a subsequence (a_{n_k}) of (a_n) such that $\lim_{k\to\infty} a_{n_k} = +\infty$ or $\lim_{k\to\infty} a_{n_k} = -\infty$.

2. Suppose (a_n) is a sequence such that $\lim_{n\to\infty} a_n = +\infty$. Prove that for any subsequence (a_{n_k}) of (a_n), $\lim_{k\to\infty} a_{n_k} = +\infty$.

3. Do # 2.5.5

4. Consider $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$. Let $S_n = \sum_{k=1}^{n} \frac{(-1)^{n+1}}{n}$.

a) Prove that the sequence $S_1, S_3, \ldots, S_{2k+1}, \ldots$ is decreasing and bounded below by 0.
b) Prove that the sequence $S_2, S_4, \ldots, S_{2k}, \ldots$ is increasing and is bounded from above by 1.

c) Prove that $\lim_{k\to\infty} S_{2k+1}$ and $\lim_{k\to\infty} S_{2k}$ exist and are equal.
d) Prove that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges.