Homework # 14.

- Read pp. 56 - 64.

- Need to know: Definition of convergence of series, Cauchy Condensation test (with proof), definition of subsequence.

<u>Do the following problems:</u>

1. For each sequence (a_n) below find the sequences $y_n = \sup\{a_k : k \geq n\}$ and $z_n = \inf\{a_k : k \geq n\}$. Also compute $\limsup a_n$ and $\liminf a_n$. (No proofs necessary).

a) $a_n = (-1)^n$, $n \in \mathbb{N}$.

b) $a_n = \sin\left(\frac{n\pi}{6}\right)$, $n \in \mathbb{N}$.

c) $a_n = (-1)^n \frac{n}{n+1}$, $n \in \mathbb{N}$.

2. Apply Cauchy Condensation test to the series $\sum_{n=2}^{\infty} \frac{1}{n \cdot (\ln n)^p}$ and decide for which values of p this series converges.

3. For each series, find an explicit formula for the sequence of partial sums and determine if the series converges.
(a) $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$, (b) $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$, c) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.

4. # 2.4.5 (a)